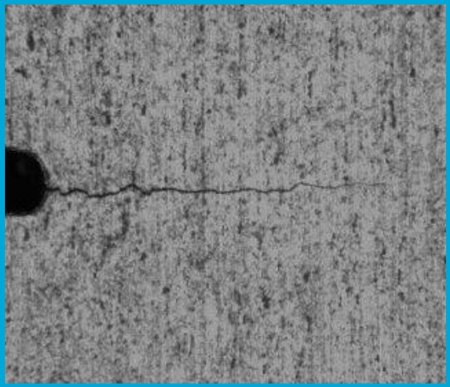
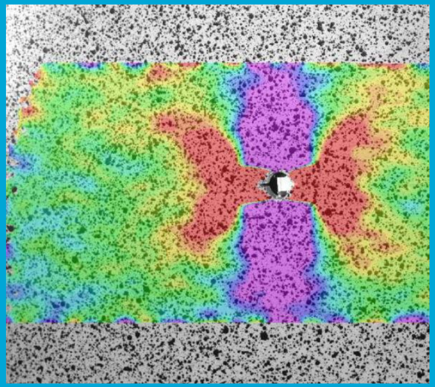
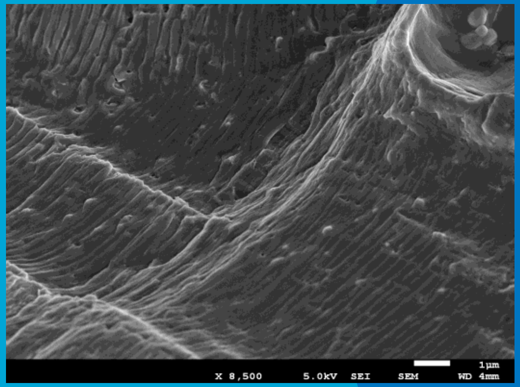






Can we predict fatigue crack growth without the help of Paris?

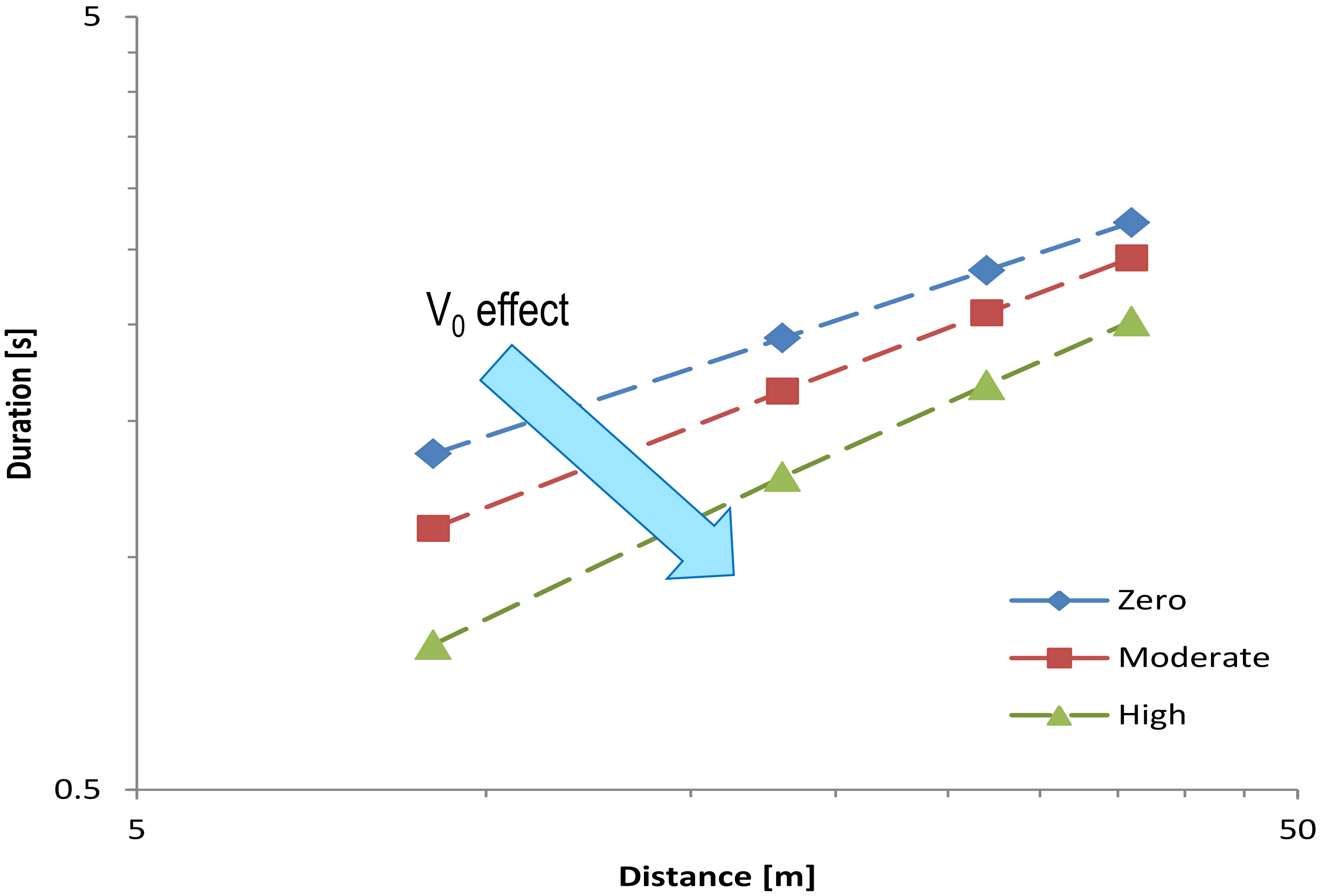
René Alderliesten / 29 June 2023

TUDelft

Philosophical Introduction

No.	Floor	Initial velocity	Time [s]
1	12	None	2.71
2	12	Moderate	2.44
3	12	High	2.02

Philosophical Introduction



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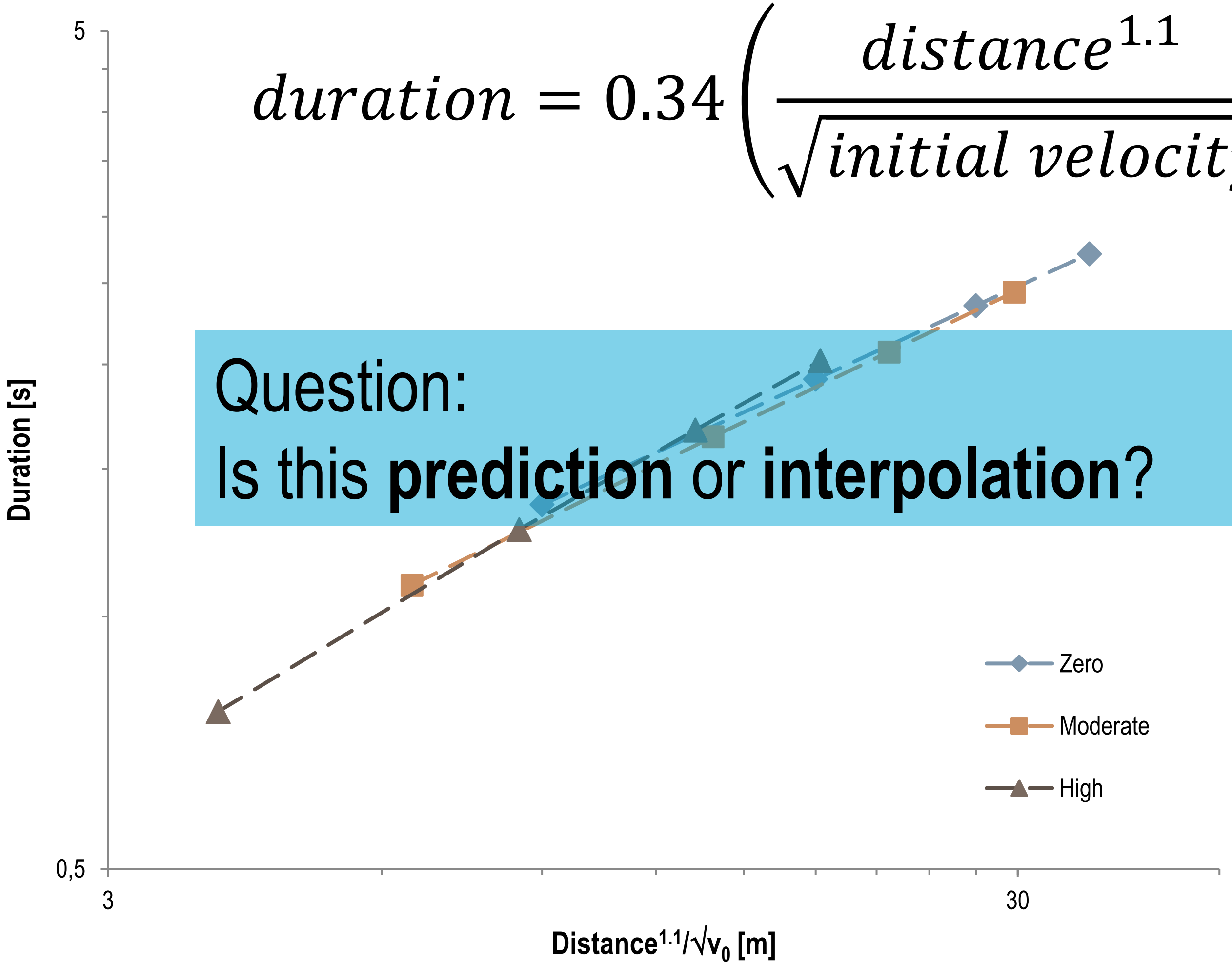
Philosophical Introduction

$$duration = 0.34\left(\frac{distance^{1.1}}{\sqrt{initial\ velocity}}\right)^{0.6}$$

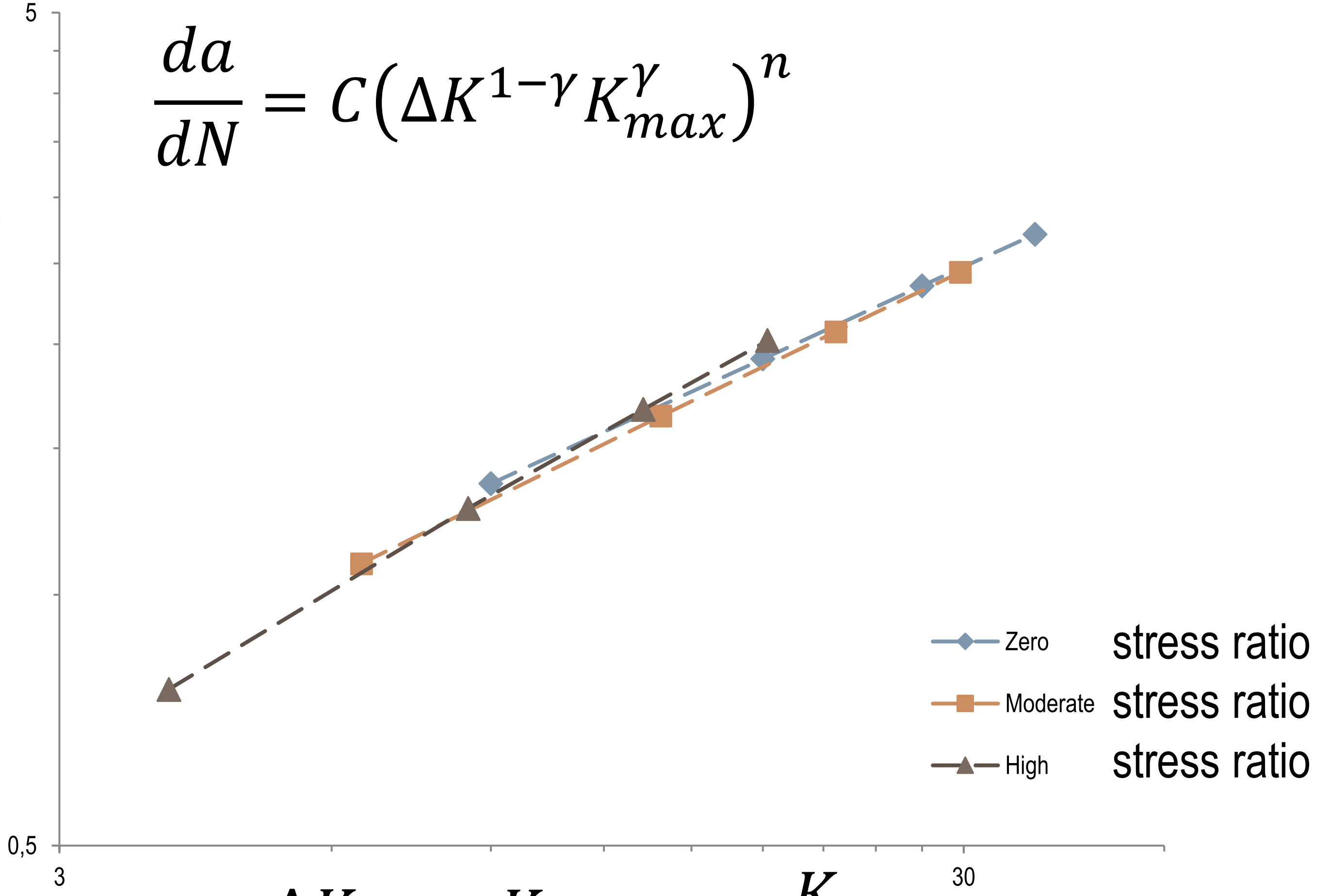


Question:
Is this **prediction** or **interpolation**?

Experimentalists…

$$\frac{da}{dN} = C\left(\Delta K^{1-\gamma} K_{max}^{\gamma}\right)^{n}$$

$\frac{da}{dN}$

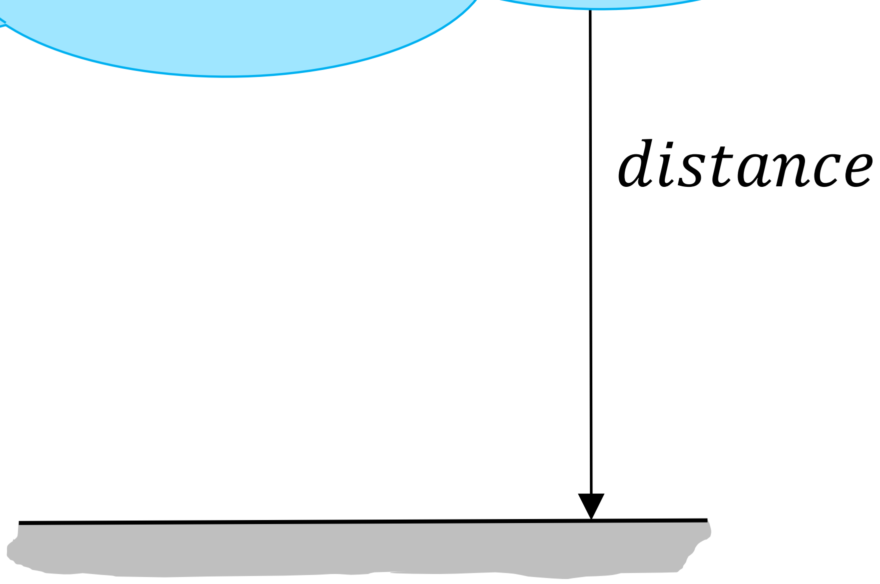


ΔK or K_{max} or $\frac{K_{max}}{K_{Ic}}$ or…

Physicists…

$$duration = t = 0.34\left(\frac{distance^{1.1}}{\sqrt{initial\ velocity}}\right)^{0.6}$$

$$duration = \frac{1}{g}\left[\sqrt{v_0^2 + 2g \cdot distance} - v_0\right]$$



$$velocity(t) = v_0 + g\,t$$

$$distance(t) = v_0\,t + \tfrac{1}{2}\,g\,t^2$$

Problem statement

I **need to test** multiple cases **before I can predict**…

Hence: I need data first, before I predict that data.

- Equivalent to AI / Neural Network concepts that need data before predictions can be made?

Can I predict without prior fatigue crack growth data?

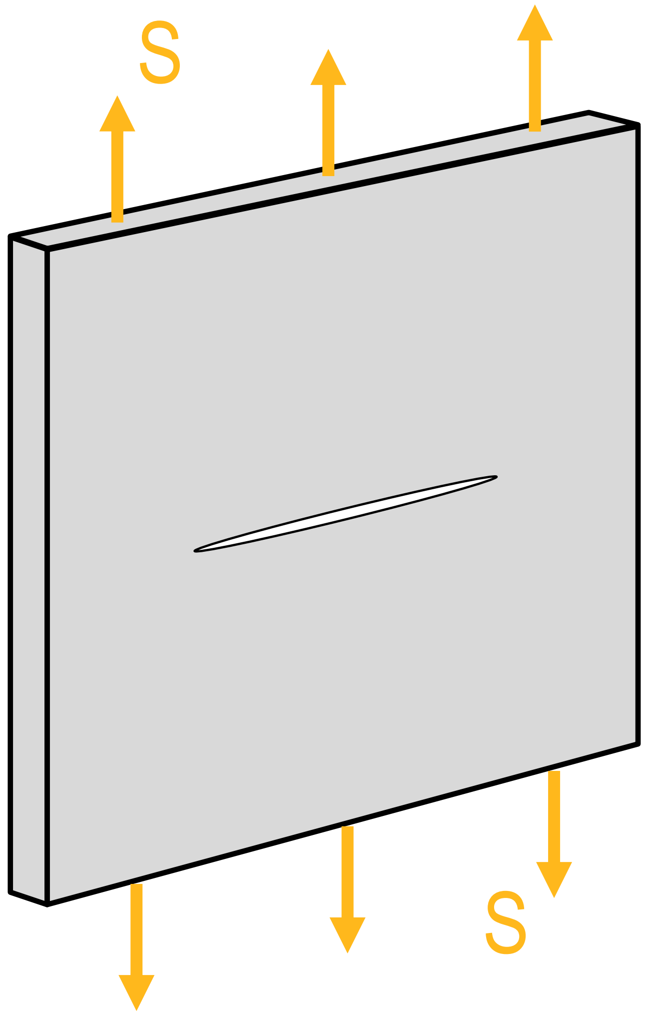
Fundamental physics

At any point in time, there is energy conservation

$$\dot{W} = \dot{U}_a + \dot{U}_p + \dot{U}_e$$

where $\dot{U}$ denotes derivative, i.e. $\frac{dU}{dt}$

Or through integration over entire load cycle using the applied frequency $f = \frac{N}{\Delta t}$ in discrete form $\frac{dU}{dN}$



Fundamental physics

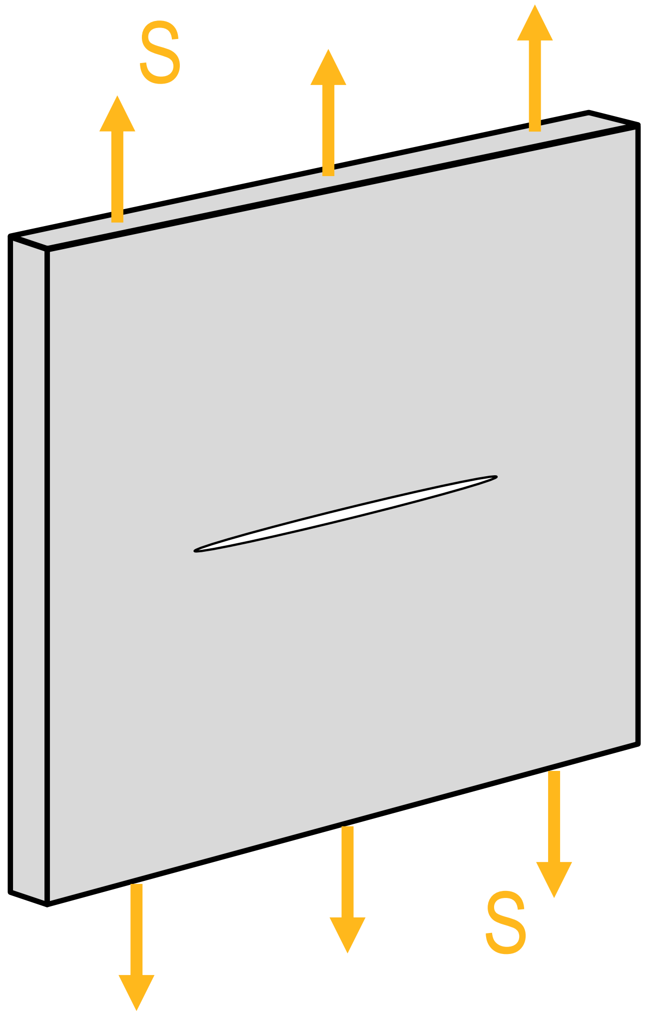
Discrete energy balance

$$\frac{dW}{dN} = \frac{dU}{dA}\frac{dA}{dN}$$

describes physics, does not constitute prediction

Physics: describes the process

Engineering prediction: how many times can I repeat that process?



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Let’s try to predict

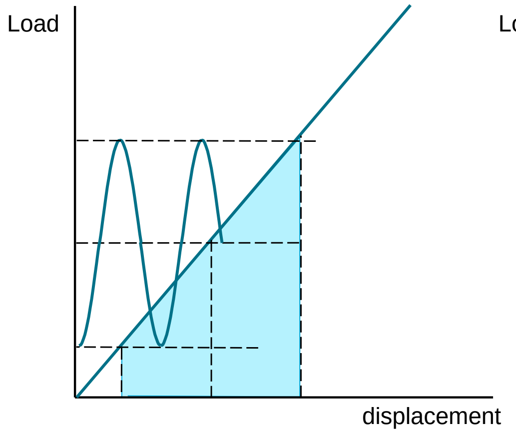
Methodology for load application

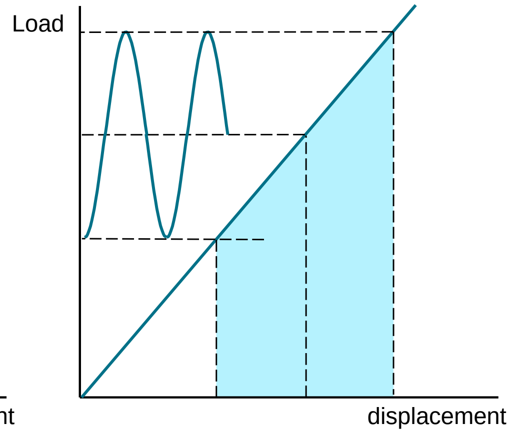
Fatigue loading implies application of Work

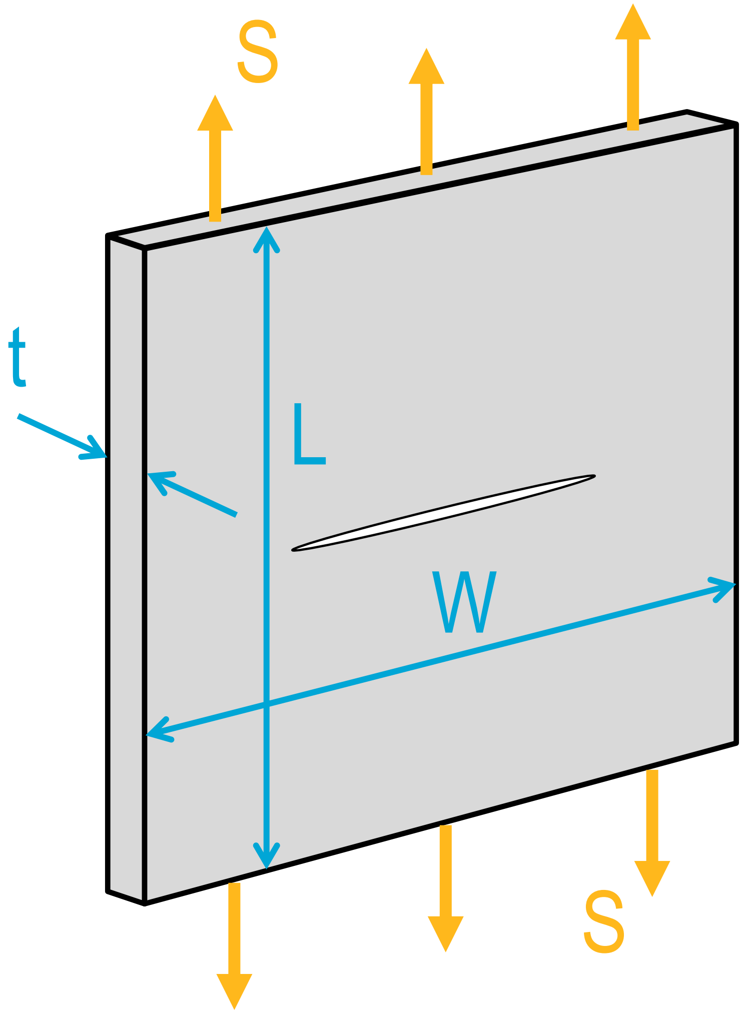
$$U_0 = \frac{1}{2} S\varepsilon WLt = \frac{S^2}{2E} WLt$$

In case of cyclic loading:

$$U_{cyc} = \frac{S_{max}^2 - S_{min}^2}{2E} WLt$$



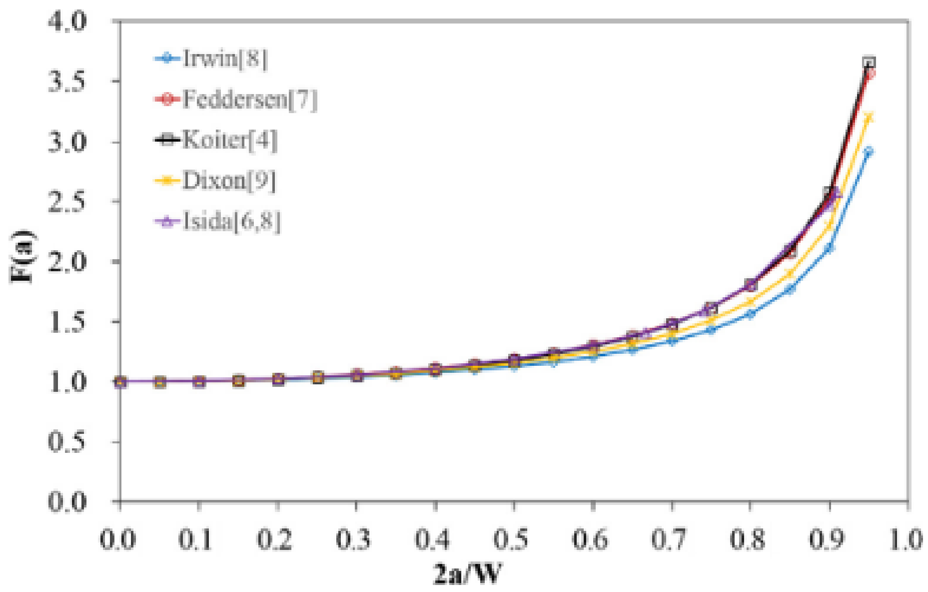




Methodology

Plate containing crack with length *2a*

- Compliance increases with *a*
- Stiffness decrease under constant load implies work increase



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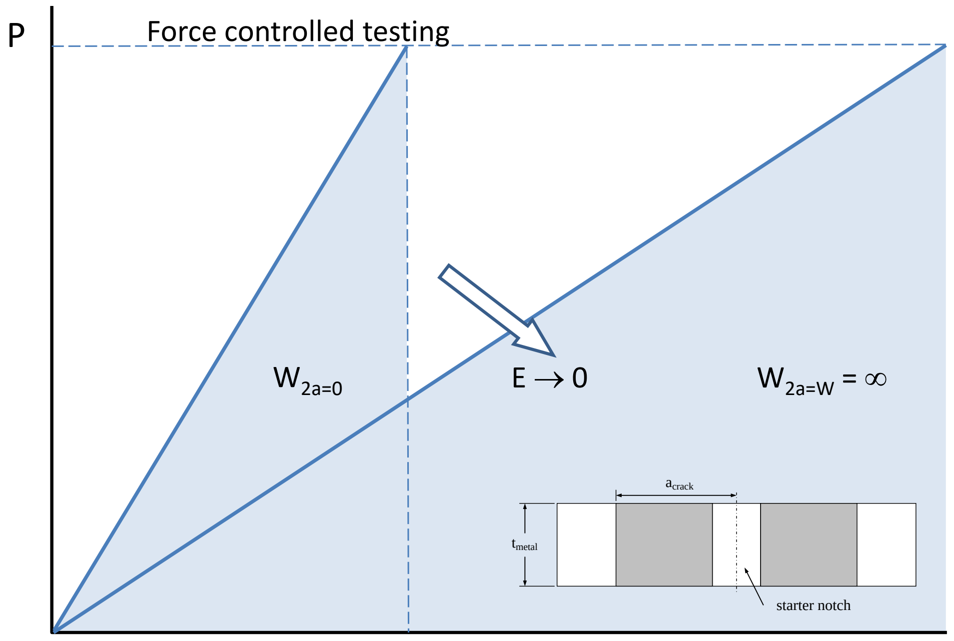


Alderliesten, Springer 2017

$$\beta = F(a) = \frac{U_a}{U_0} = \sqrt{sec\left(\frac{\pi a}{W}\right)}$$

Methodology for load application

Fatigue loading implies application of Work

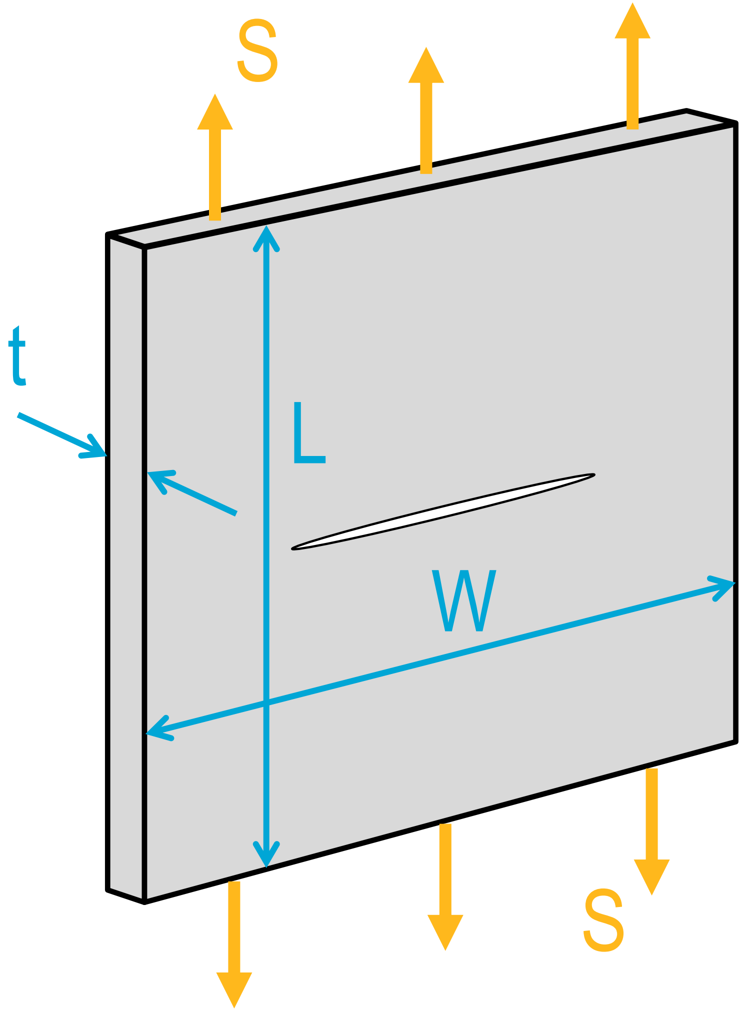
$$U_0 = \frac{1}{2} S\varepsilon WLt = \frac{S^2}{2E} WLt$$

In case of cyclic loading:

$$U_{cyc} = \frac{S_{max}^2 - S_{min}^2}{2E} WLt$$

In case of finite width:

$$U_{cyc,a} = \frac{S_{max}^2 - S_{min}^2}{2E} WLt \sqrt{sec\left(\frac{\pi a}{W}\right)}$$



Note

The energy balance

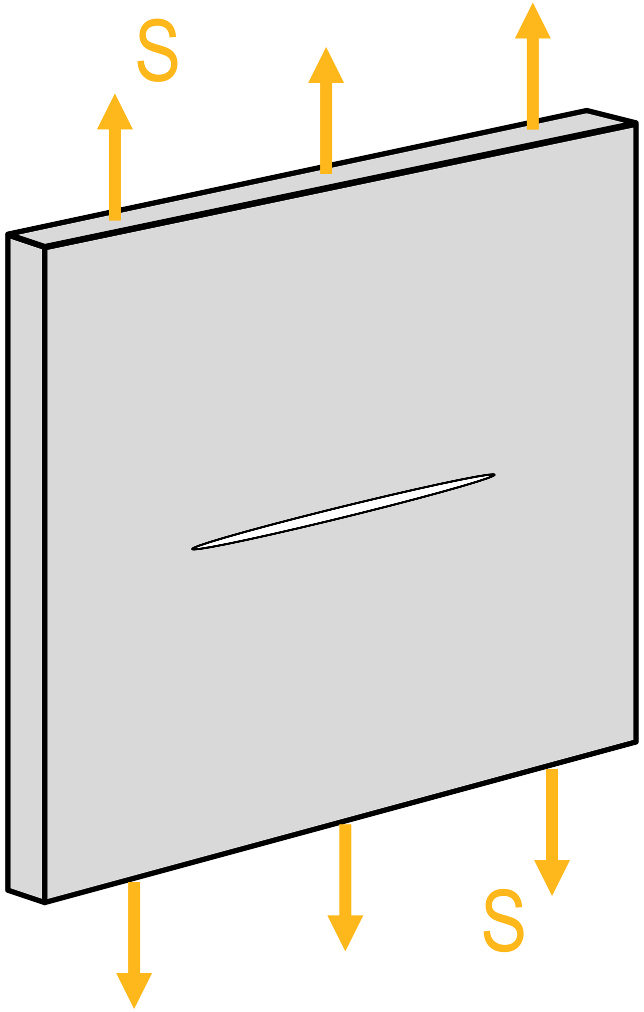
$$\frac{dW}{dN} = \frac{dU}{dA}\frac{dA}{dN}$$

Does not relate dW to W ….

Let’s look at the energy balance first

$$\dot{W} = \dot{U}_a + \dot{U}_p + \dot{U}_e$$

$$\frac{dW}{dN} = \frac{dU_a}{da}\frac{da}{dN} + \frac{dU_p}{dV_p}\frac{dV_p}{dN}$$



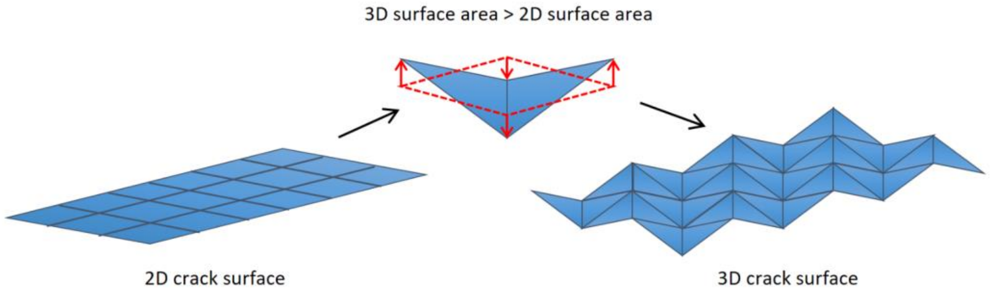
Fracture surface formation

Fracture surface energy dissipation

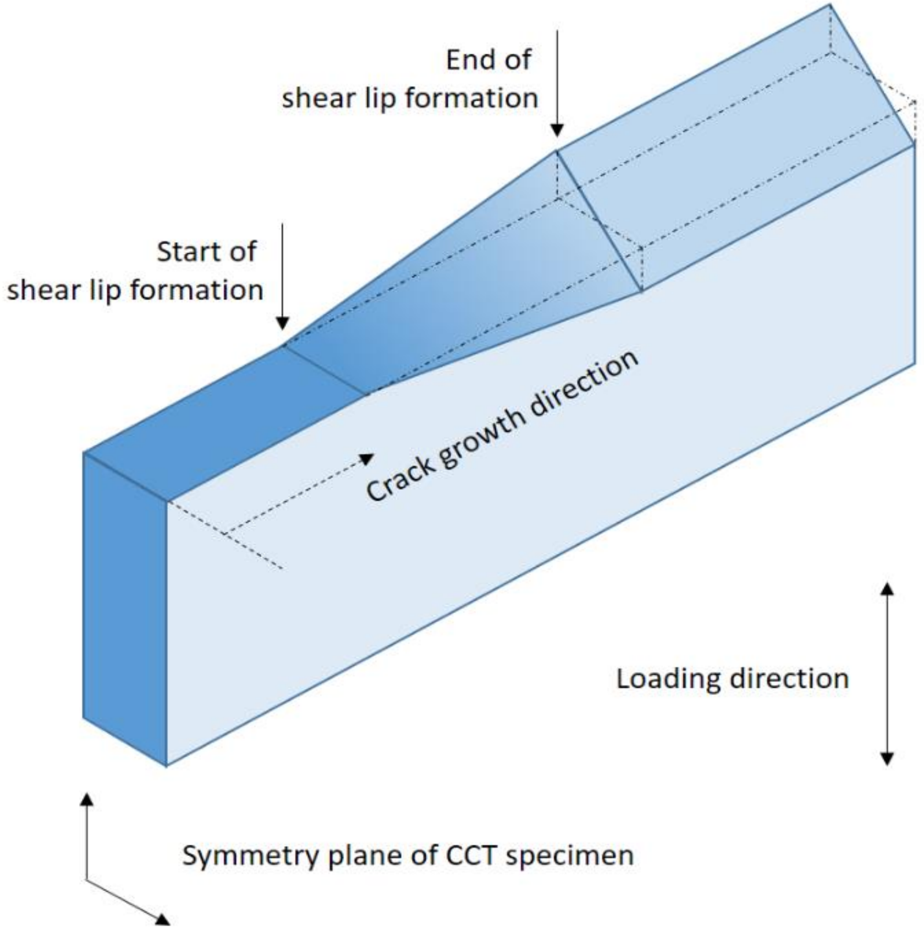
$$\frac{dU_a}{dN} = \frac{dU_a}{dA} t\gamma\lambda \frac{da}{dN}$$

where $\frac{dU_a}{dA}$ is surface energy, ~ 2 J/m^2

$$\lambda = \frac{r_a(a)}{r_a(0)}$$



$\lambda \sim \sqrt{2}$

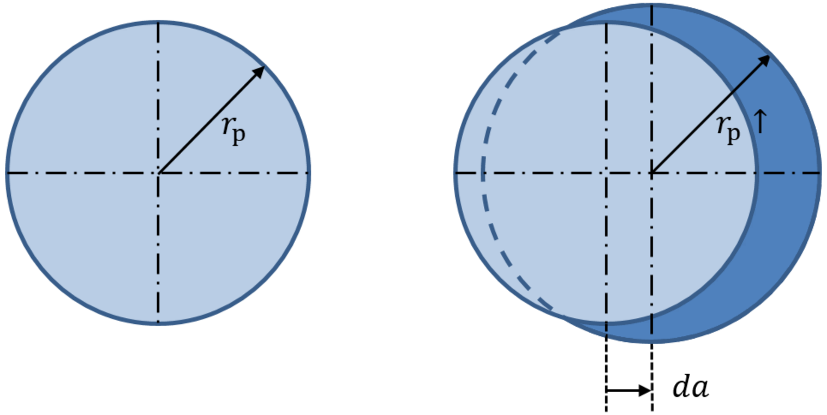


Plasticity (volume)

Irwin's plastic zone approximation for plastic volume

$$r_p = \frac{1}{\alpha\pi}\left(\frac{K_{max}}{S_{yield}}\right)^2$$

with α=1 for plane stress, α=2 for plane strain



Plastic volume (cylinder in thickness direction)

$$V_p = \pi r_p^2 t = \frac{\pi}{\alpha^2} a^2 sec^2\left(\frac{\pi a}{W}\right)\left(\frac{S_{max}}{S_{yield}}\right)^4 \quad \rightarrow V_p = f(a) \quad \rightarrow \frac{dV_p}{dN} = f\left(\frac{da}{dN}\right)$$

grows as crescent volume due to crack tip translation *da* and increase in radius r_p

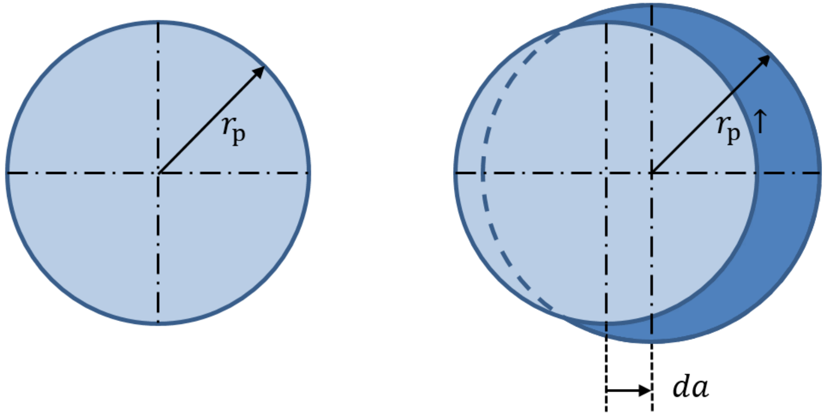
Plasticity (energy density)

Plastic energy dissipation

$$\frac{dU_p}{dN} = \frac{dU_p}{dV_p}\frac{dV_p}{dN}$$

Crescent volume increase

Mean energy density



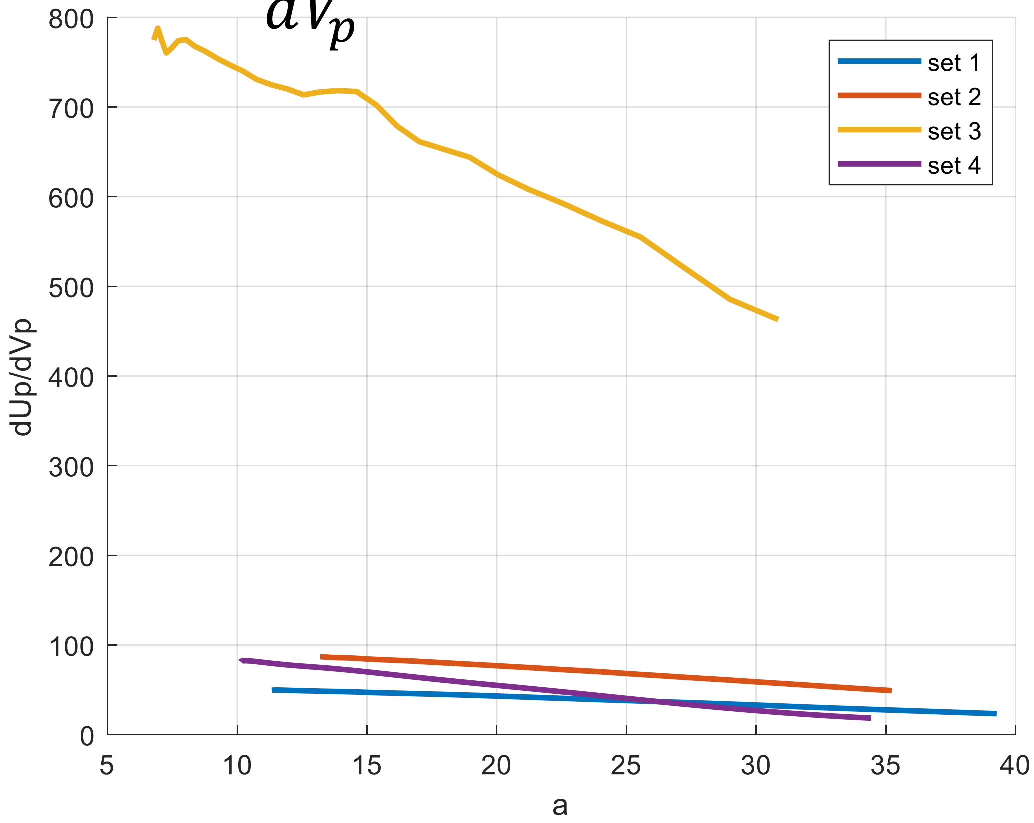
Solve energy balance on tests with dU_p/dV_p as the only unknown

Test	Material	W [mm]	t [mm]	a_0 [mm]	S_{max} [MPa]	R	S_{yield} [MPa]	Ref
1	2024-T3	160	6.1	13.18	60	0	324	[7]
2	2024-T3	160	6.1	11.25	80	0.3	324	[7]
3	7075-T6	160	3.2	6.35	48.8	0.1	503	[5,9]
4	Fe510Nb	100	10	10.0	113	0.5	460	[10]

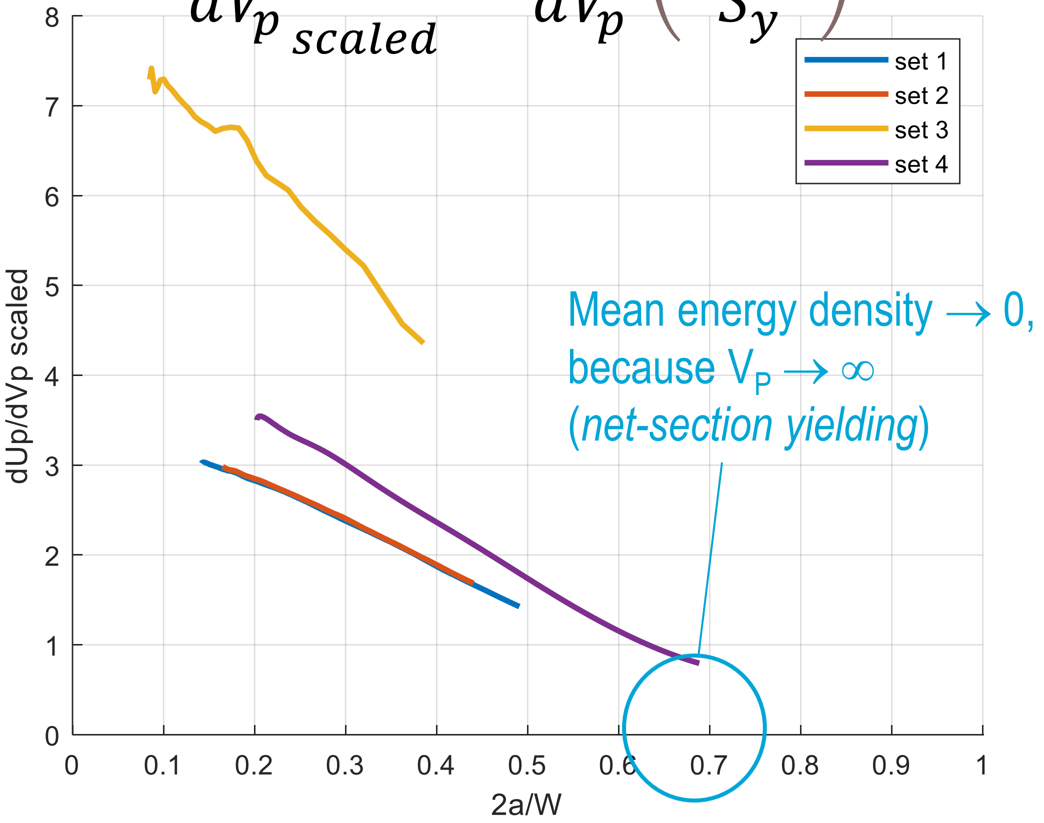
Plasticity (energy density) – con'd

Mean plastic energy density through solving energy balance

$$\frac{dU_p}{dV_p}$$



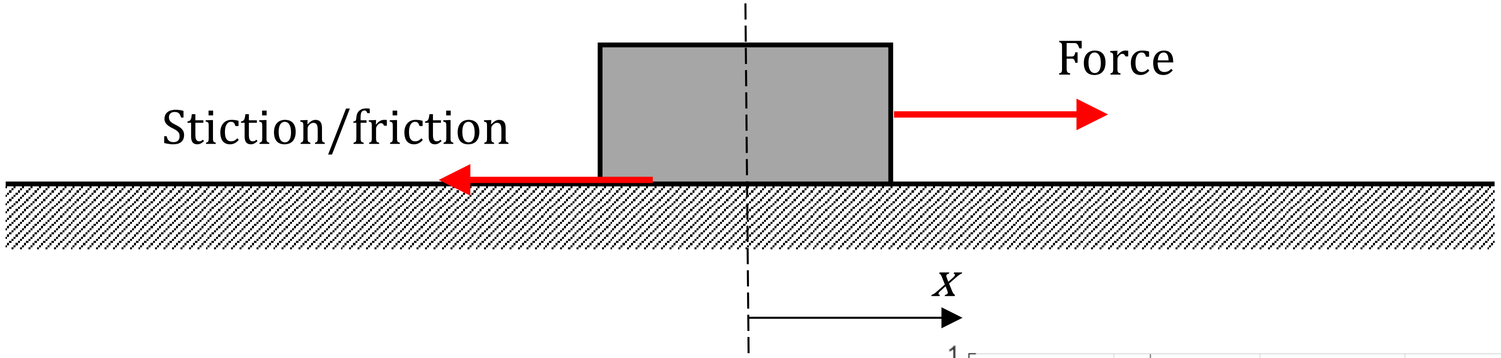
$$\left.\frac{dU_p}{dV_p}\right._{scaled} = \frac{dU_p}{dV_p}\left(\frac{S_{max}}{S_y}\right)^2$$



Let's now relate W to dW…

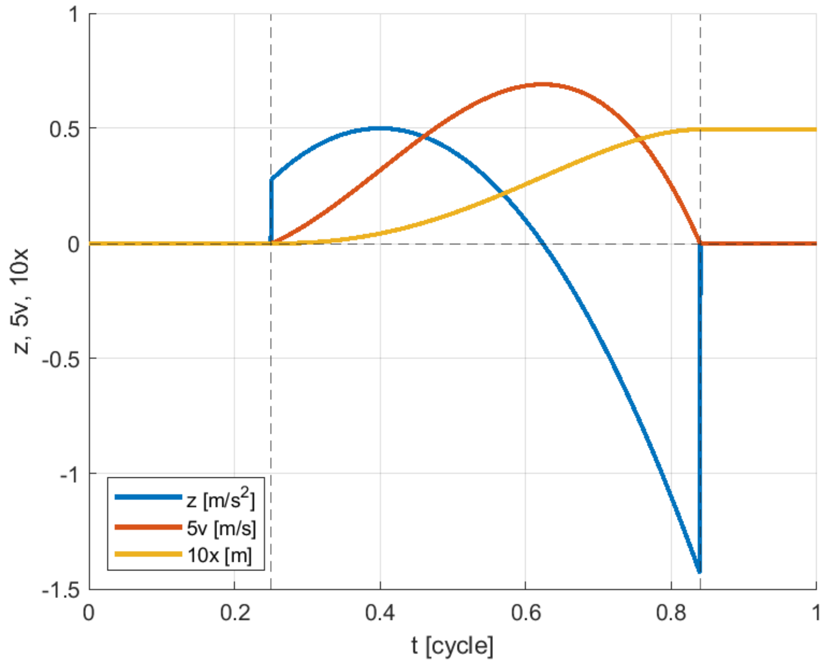
Second concept from physics

$$z(t) = \frac{F_{net}(t)}{M} = \frac{F(t) - F_f}{M}$$

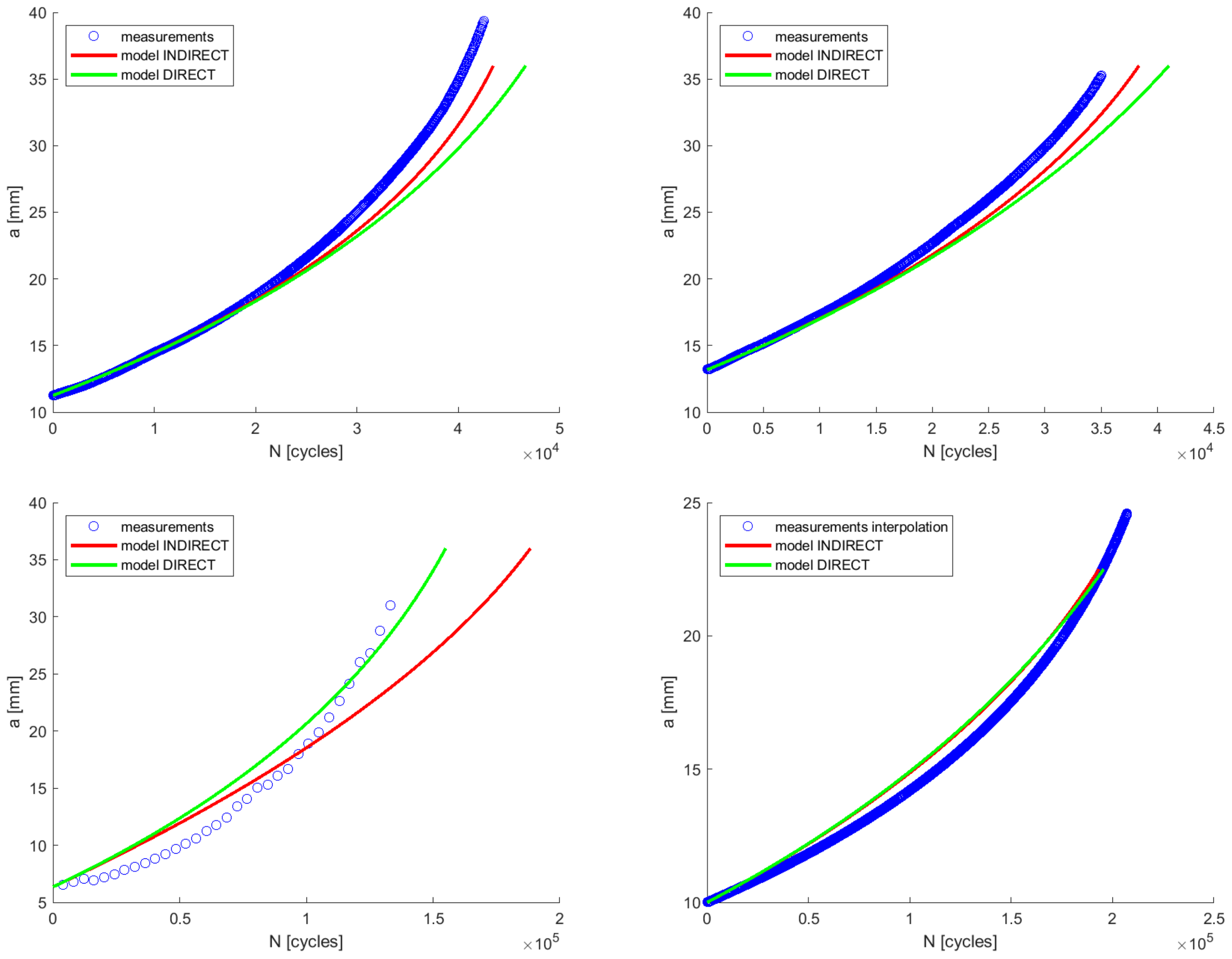


$$v(t) = v_0 + z(t)t$$

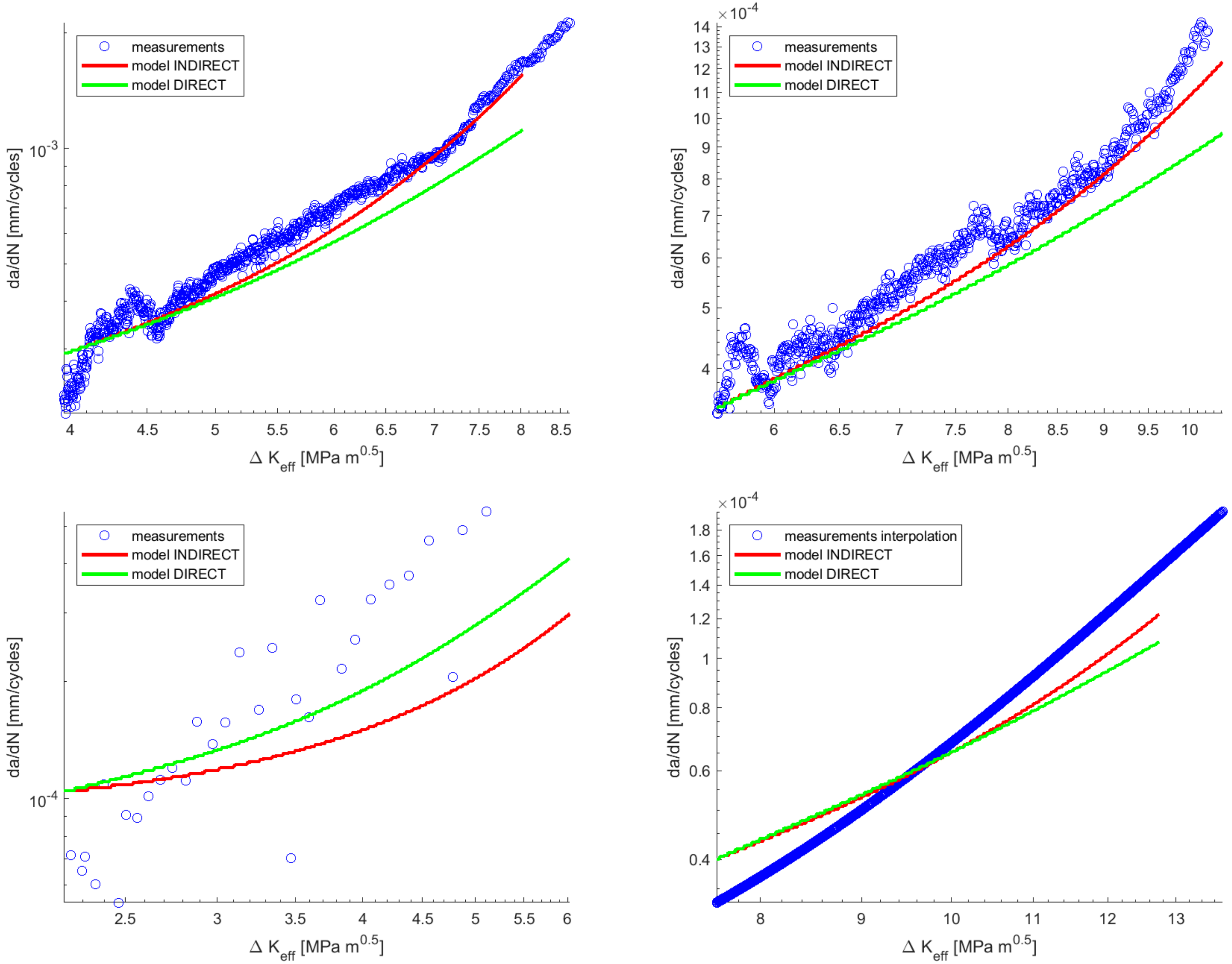
$$\mathrm{x}(t) = x_0 + v(t)t + z(t)t^2$$



Result


measurements
model INDIRECT
model DIRECT
measurements interpolation
a [mm]
N [cycles]
$\times 10^4$
$\times 10^5$

Result


measurements
model INDIRECT
model DIRECT
da/dN [mm/cycles]
Δ K_eff [MPa m^0.5]
measurements interpolation
×10^-4

Discussion

One may argue that this method assumes little influence of material microstructure

However:

- it’s implicitly in the stress-strain relationship (to be further developed!)
- literature reveals that microstructure indeed may have limited influence

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Conclusion

Can we predict fatigue crack growth without the help of Paris?

Yes, we can!

Thank you for your attention

René Alderliesten

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