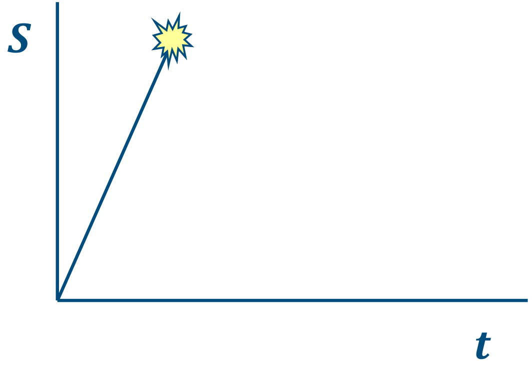
Dedicated to innovation in aerospace

Back to basics for the fatigue crack growth rate in metallic alloys

E. Amsterdam, J.W.E. Wiegman, M. Nawijn, J.Th.M. De Hosson

ICAF2023 Delft 2023

Brittle



Crack extension occurs when:

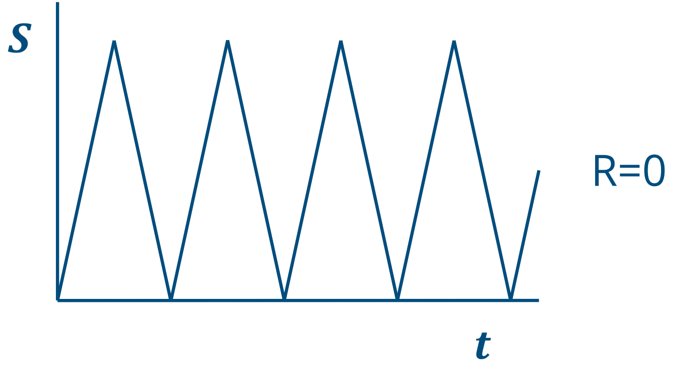
$$G_{max} = G_C = \frac{K_{max}^2}{E}$$

G = Classical **crack driving force** in brittle materials

Griffith (1920)

Irwin (1957)

Ductile

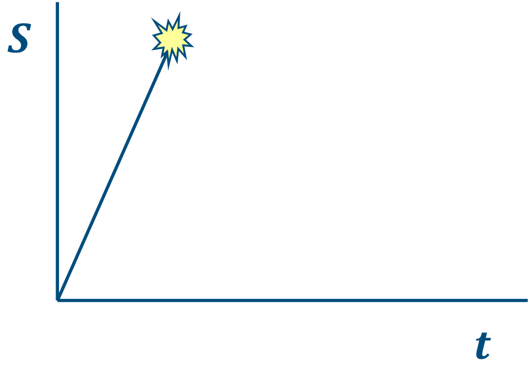


Crack extension under cyclic loading:

$$\frac{da}{dN} = C\left(\frac{K_{max}}{E}\right)^n$$

Frost and Greenan (1964), Pearson (1966)

Brittle



Crack extension occurs when:

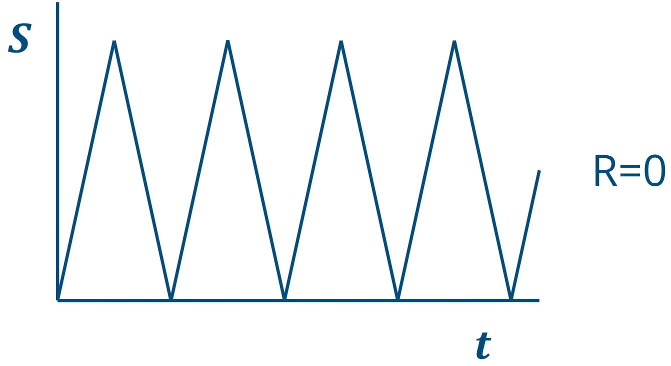
$$G_{max} = G_C = \frac{K_{max}^2}{E}$$

G = Classical **crack driving force** in brittle materials

Griffith (1920)

Irwin (1957)

Ductile



Crack extension under cyclic loading:

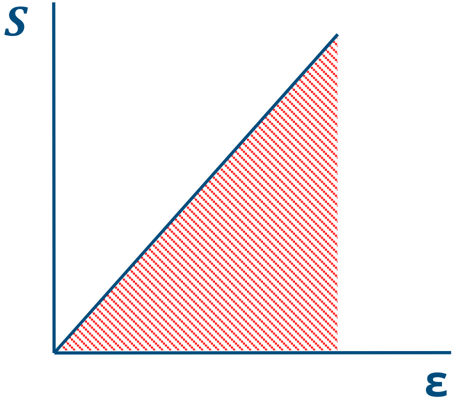
$$\frac{da}{dN} = C\left(\frac{K_{max}}{E}\right)^n$$

$$= C\left(\frac{K_{max}}{K_{max}}\frac{K_{max}}{E}\right)^n \quad = C\left(\frac{G_{max}}{K_{max}}\right)^n$$

driving force (G_{max})

plasticity (K_{max})

R>0

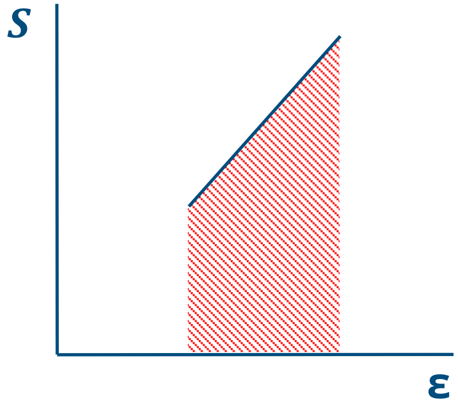


R=0

$$\frac{da}{dN} = C\left(\frac{G_{max}}{K_{max}}\right)^n$$

$$G_{max} = \frac{S_{max}^2 \pi a}{E} = \frac{S_{max}^2}{2E} \cdot 2\pi a$$

∝ strain energy density; U′

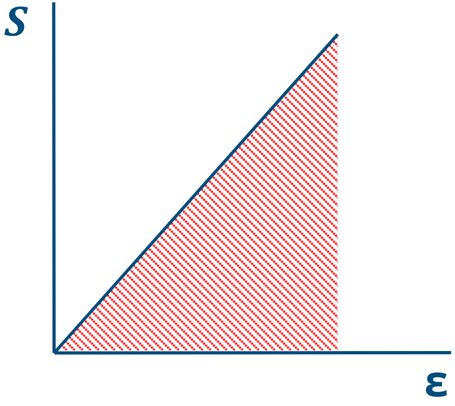


Cyclic strain energy density: $\Delta U' = \frac{S_{max}^2}{2E} - \frac{S_{min}^2}{2E}$

R≥0 => $\frac{da}{dN} = C\left(\frac{G_{max} - G_{min}}{K_{max}}\right)^n$

 emiel.amsterdam@nlr.nl

R>0

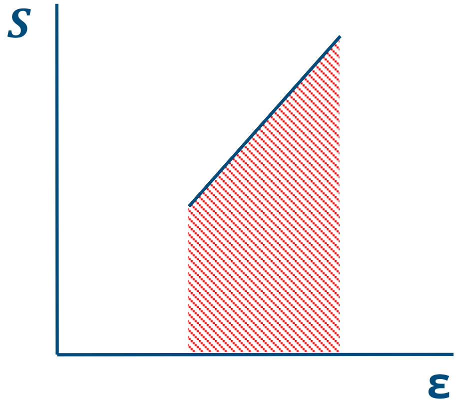


R=0

$$\frac{da}{dN} = C\left(\frac{G_{max}}{K_{max}}\right)^n$$

$$G_{max} = \frac{S_{max}^2 \pi a}{E} = \frac{S_{max}^2}{2E} \cdot 2\pi a$$

∝ strain energy density; U'



Cyclic strain energy density: $\Delta U' = \frac{S_{max}^2}{2E} - \frac{S_{min}^2}{2E}$

R≥0 => $\frac{da}{dN} = C\left(\frac{\Delta G}{K_{max}}\right)^n$

emiel.amsterdam@nlr.nl

New equation

$$\frac{da}{dN} = C\left(\frac{\Delta G}{K_{max}}\right)^n$$

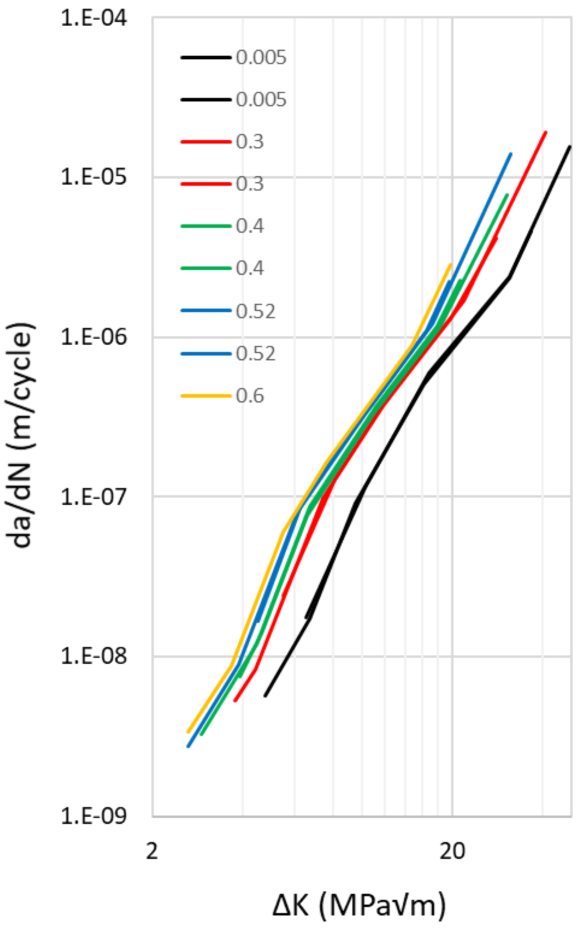
$$= C\left(\frac{(1-R^2)K_{max}}{E}\right)^n$$

$$= C\left(\frac{(1+R)(1-R)K_{max}}{E}\right)^n$$

$$= C^*\left(\frac{(0.5+0.5R)\cdot\Delta K}{E}\right)^n$$

$$\approx C^*\left(\frac{(0.5+0.4R)\cdot\Delta K}{E}\right)^n$$

$$\approx C^*\left(\frac{\Delta K_{eff}}{E}\right)^n$$



 emiel.amsterdam@nlr.nl

New equation

$$\frac{da}{dN} = C\left(\frac{\Delta G}{K_{max}}\right)^n$$

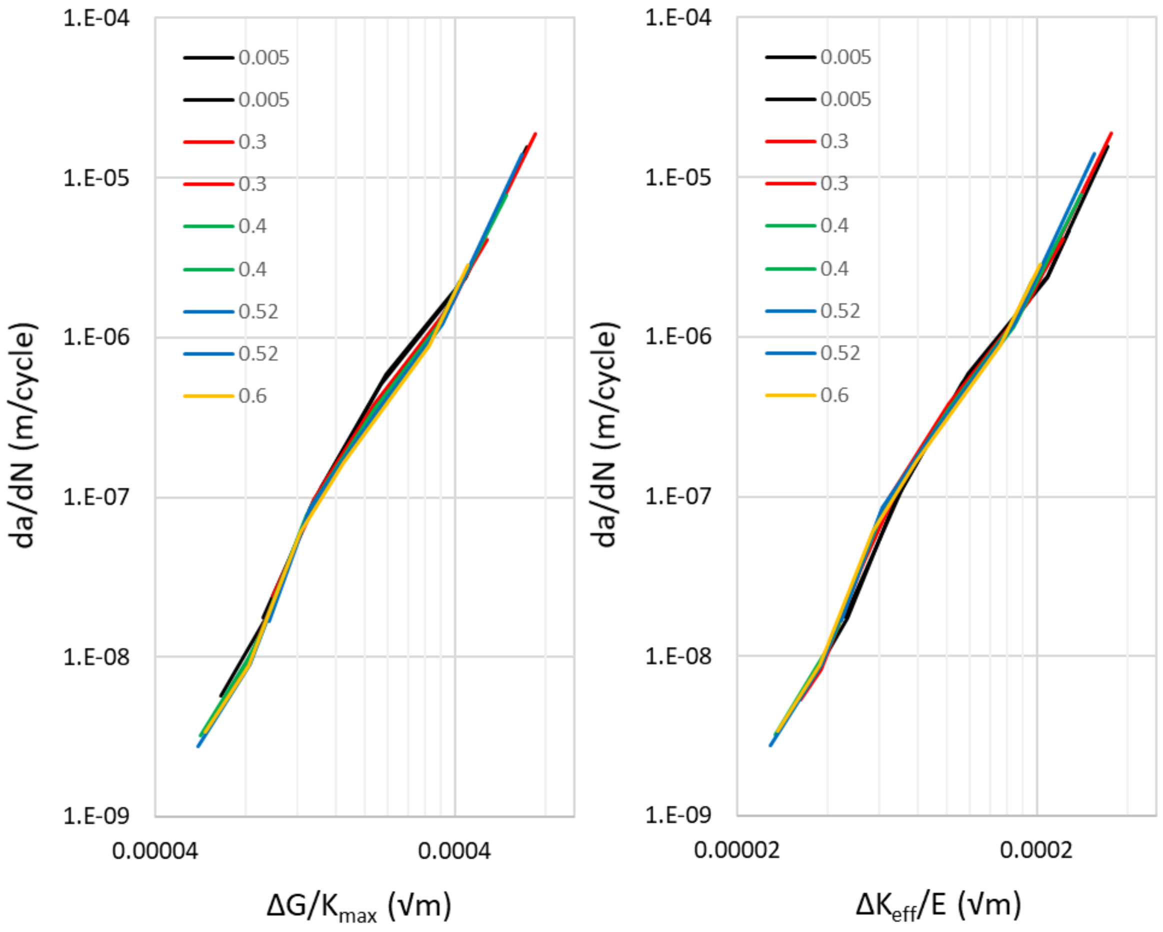
$$= C\left(\frac{(1-R^2)K_{max}}{E}\right)^n$$

$$= C\left(\frac{(1+R)(1-R)K_{max}}{E}\right)^n$$

$$= C^*\left(\frac{(0.5+0.5R)\cdot\Delta K}{E}\right)^n$$

$$\approx C^*\left(\frac{(0.5+0.4R)\cdot\Delta K}{E}\right)^n$$

$$\approx C^*\left(\frac{\Delta K_{eff}}{E}\right)^n$$



Hypothesis

$$\frac{da}{dN} = C\left(\frac{\Delta G}{K_{max}}\right)^{n}$$

How to proof this hypothesis?

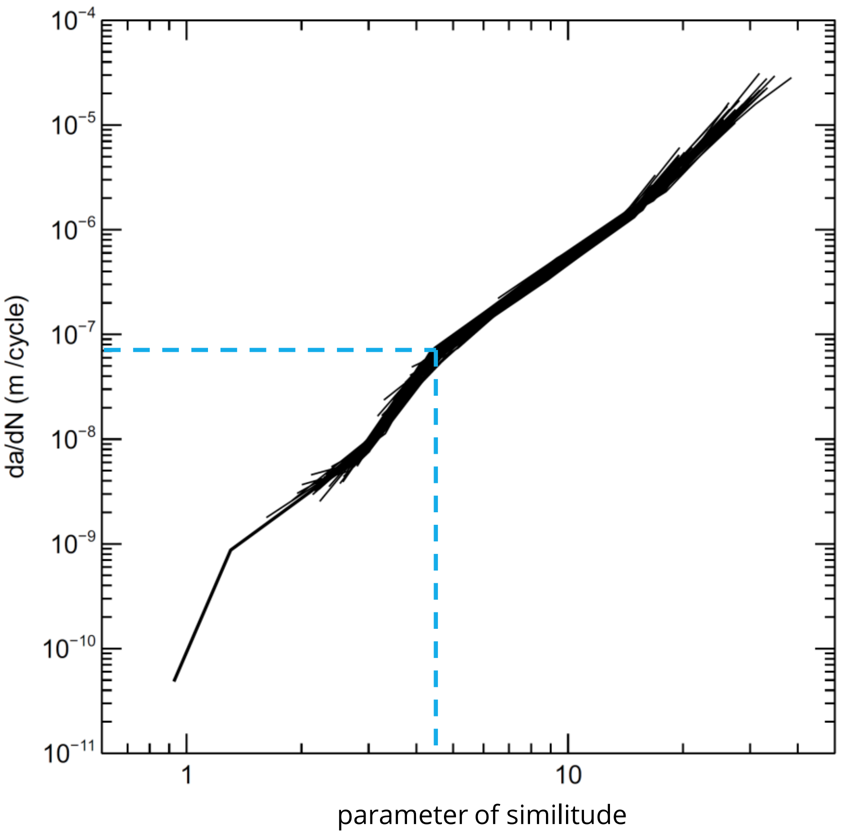
Master curve

For a given FCGR the parameter of similitude should be constant for different input parameters:

$$\frac{\Delta G}{K_{max}} = constant$$

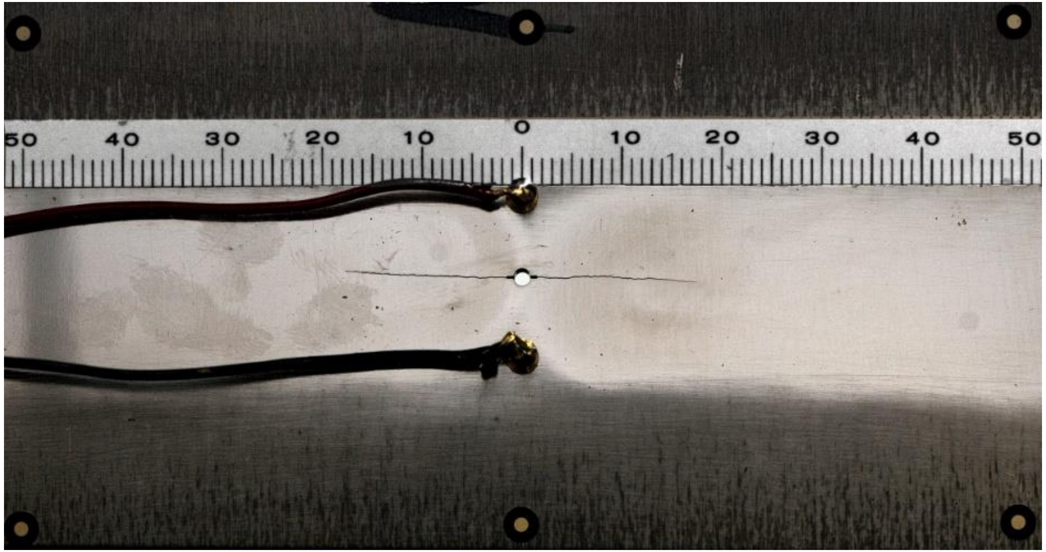
$$\Delta U' \cdot \frac{2}{S_{max}} \cdot \beta\sqrt{\pi a} = constant$$

$$\frac{1}{\beta\sqrt{\pi a}} = \frac{constant}{S_{max}} \cdot \Delta U'$$



Constant stress range fatigue crack growth rate tests on 7075-T7351

- 160 wide M(T) specimens from a single LOT
- 1.5 mm EDM notch (single side) + pre-cracking till $a_0 \approx 2$ mm with the test parameters
- Crack length measured with potential drop
- **61 specimens with various S_{max} and S_{min}**



output → a_r (crack length)

@ $6.18 \cdot 10^{-8}$ m/cycle

[E. Amsterdam, J.W.E. Wiegman, M. Nawijn, J.Th.M. De Hosson. *The effect of crack length and maximum stress on the fatigue crack growth rates of engineering alloys*. International Journal of Fatigue 161 (2022) 106919.]

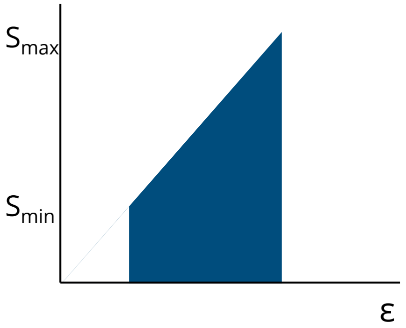
S_{max} = 100 MPa

ΔU' = cyclic strain energy density

$$= \frac{S_{max}^2}{2E} - \frac{S_{min}^2}{2E}$$

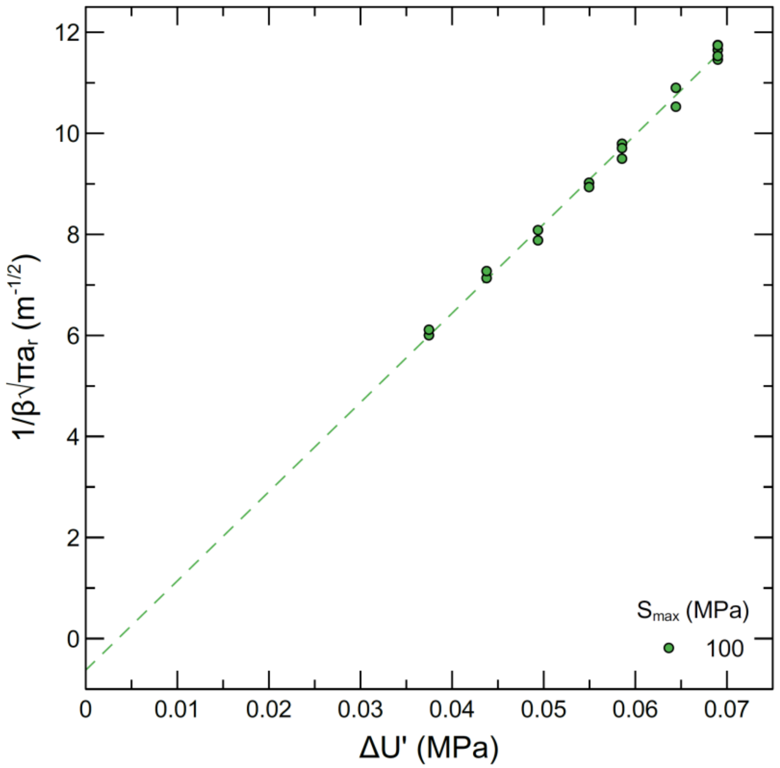
Only S_{min} is varied

and as a result R $\left(= \frac{S_{min}}{S_{max}}\right)$



R=0.3

S_{max} = 100 MPa

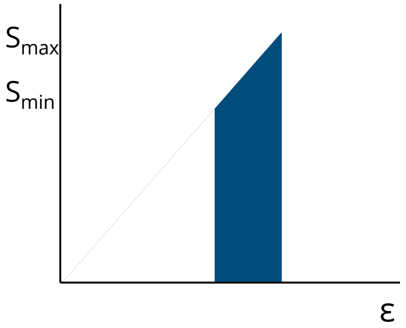


ΔU' = cyclic strain energy density

$$= \frac{S_{max}^2}{2E} - \frac{S_{min}^2}{2E}$$

Only S_{min} is varied

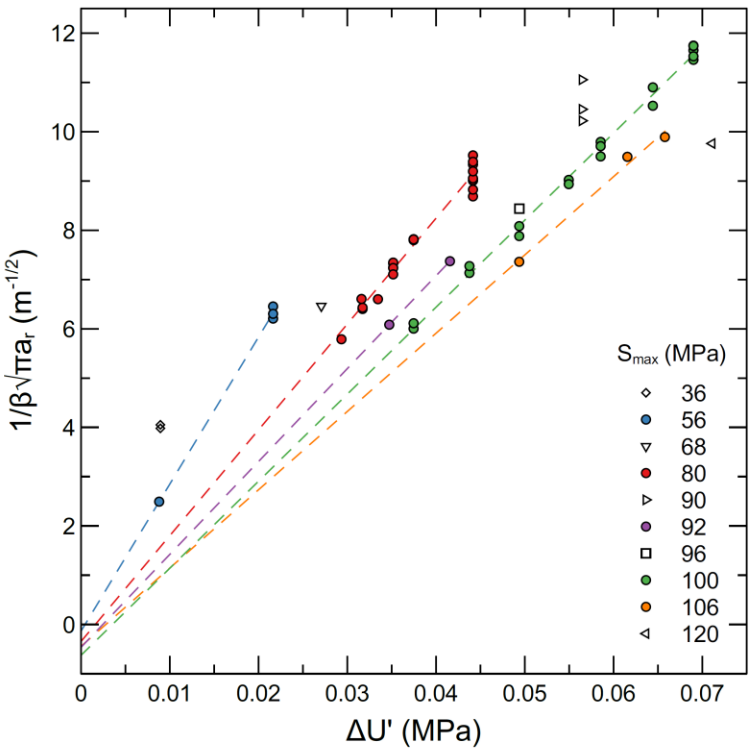
and as a result R $\left(= \frac{S_{min}}{S_{max}}\right)$

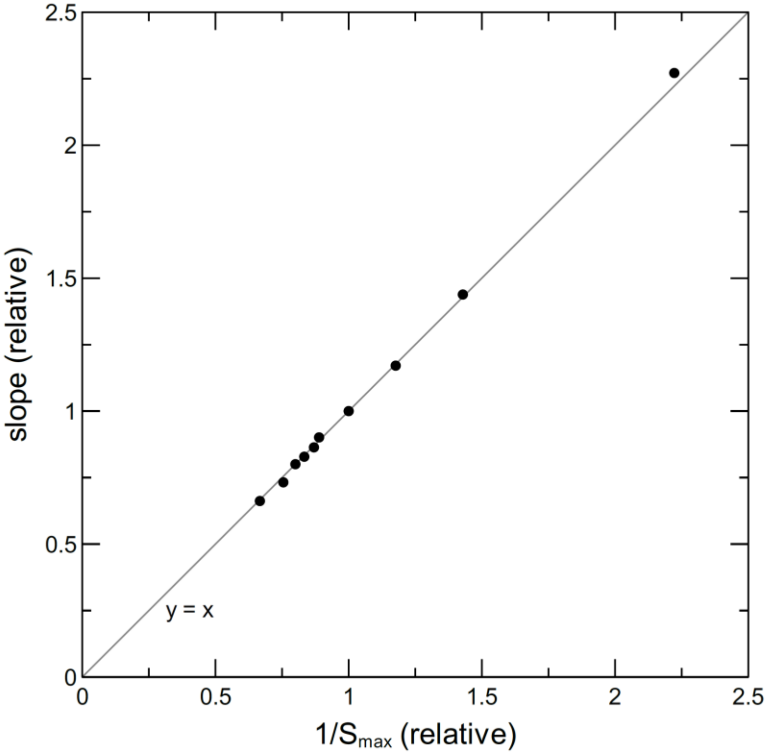


R=0.7

All results

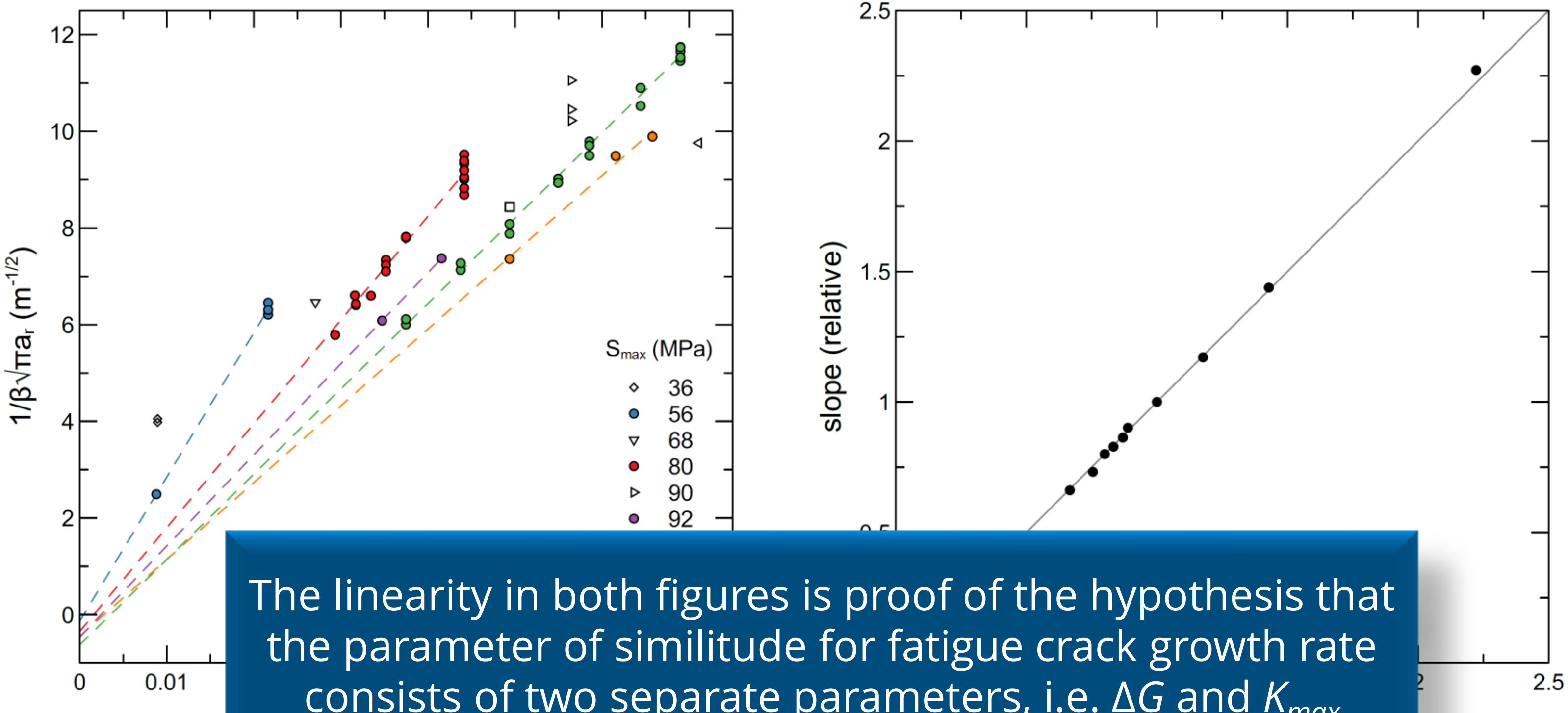
$$\frac{1}{\beta\sqrt{\pi a}} = \frac{constant}{S_{max}} \cdot \Delta U'$$



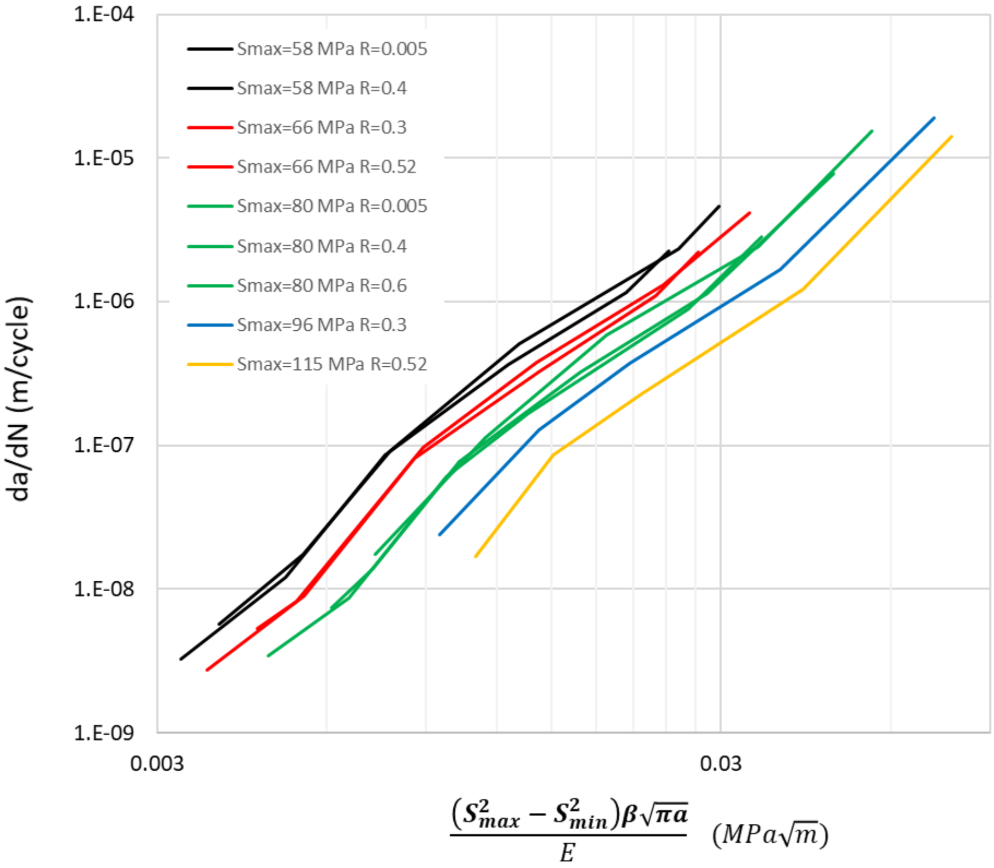


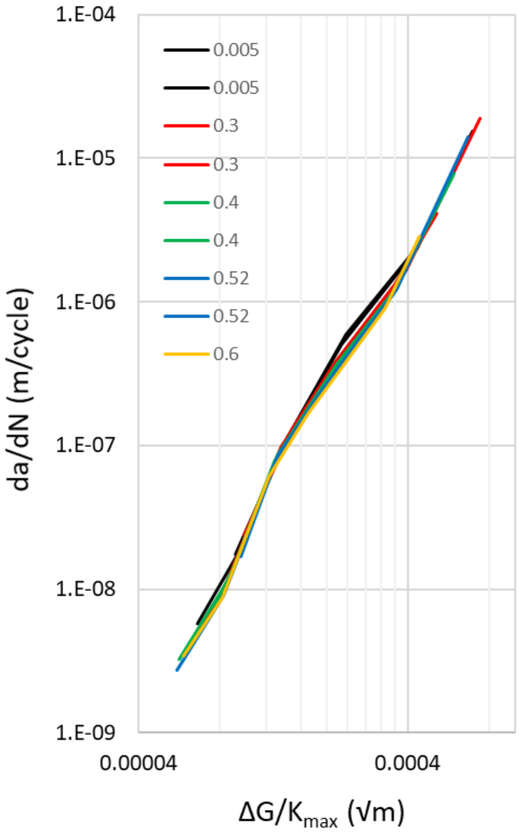
All results

$$\frac{1}{\beta\sqrt{\pi a}} = \frac{constant}{S_{max}} \cdot \Delta U'$$

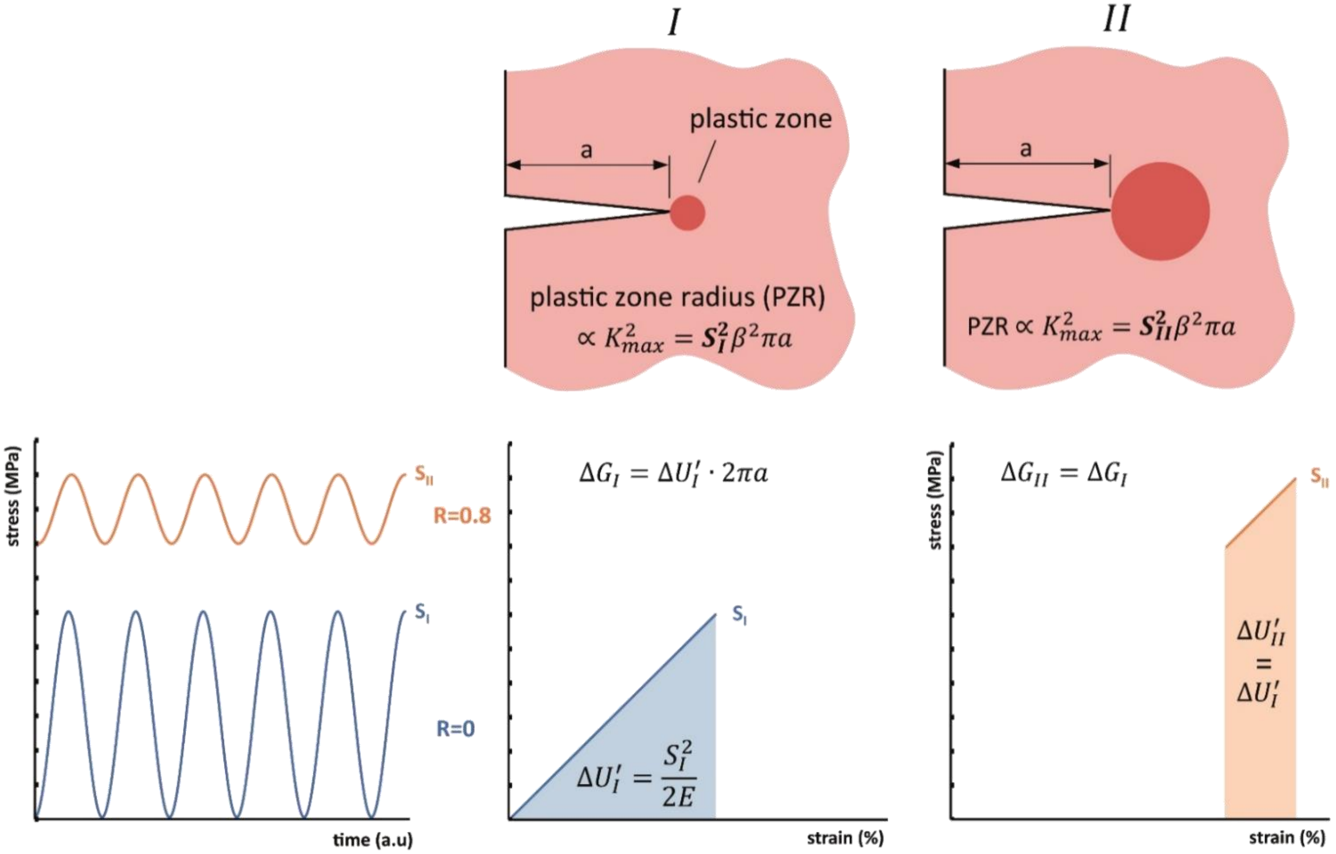


The linearity in both figures is proof of the hypothesis that the parameter of similitude for fatigue crack growth rate consists of two separate parameters, i.e. ΔG and K_{max}


1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
da/dN (m/cycle)
0.003
0.03
Smax=58 MPa R=0.005
Smax=58 MPa R=0.4
Smax=66 MPa R=0.3
Smax=66 MPa R=0.52
Smax=80 MPa R=0.005
Smax=80 MPa R=0.4
Smax=80 MPa R=0.6
Smax=96 MPa R=0.3
Smax=115 MPa R=0.52
$\frac{(S_{max}^2 - S_{min}^2)\beta\sqrt{\pi a}}{E}$ $(MPa\sqrt{m})$


1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
da/dN (m/cycle)
0.00004
0.0004
0.005
0.005
0.3
0.3
0.4
0.4
0.52
0.52
0.6
$\Delta G/K_{max}$ (√m)

The effect of plasticity



Variable amplitude (VA)

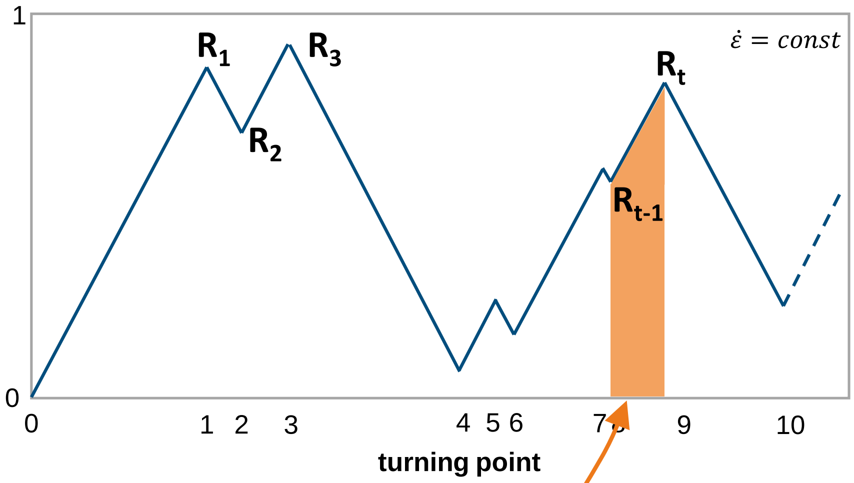
Can we use the new formulation for crack retardation and variable amplitude fatigue crack growth?

emiel.amsterdam@nlr.nl

Variable amplitude (VA)

$$R_t = \frac{S_{turning\ point}}{maximum\ spectrum\ stress\ (MSS)}$$

$$K_{ref} = \beta \cdot MSS \cdot \sqrt{\pi a}$$



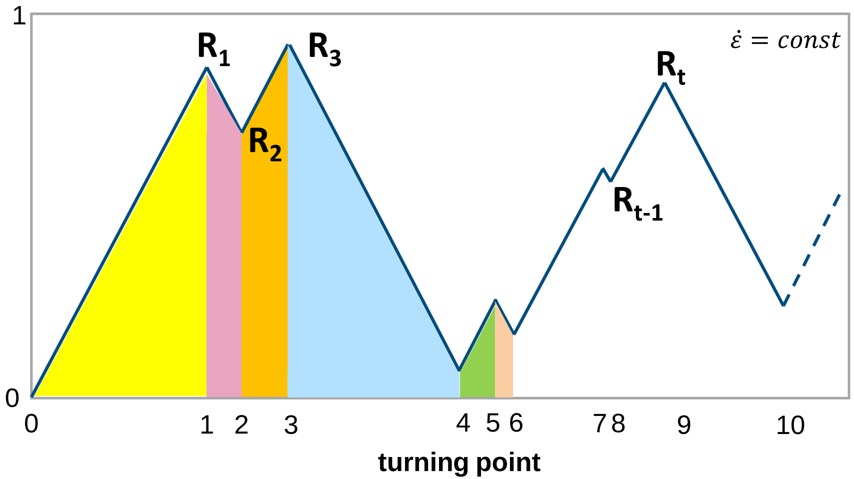
$$\frac{da}{dN} = C\left(\frac{K_{max}^2 - K_{min}^2}{E} \cdot \frac{1}{K_{max}}\right)^n = C\left(\frac{K_{max}}{E} \cdot (1 - R^2)\right)^n$$

(CA)

$$\frac{da}{dt} = C^*\left(\frac{\left|R_t^2 - R_{t-1}^2\right| K_{ref}{}^2}{E} \cdot \frac{1}{K_{ref}}\right)^n = C^*\left(\frac{K_{ref}}{E} \cdot \left|R_t^2 - R_{t-1}^2\right|\right)^n$$

(VA)

Variable amplitude (VA)



$$\frac{da}{dt} = \left|R_t^2 - R_{t-1}^2\right|^n \cdot C^* \left(\frac{K_{ref}}{E}\right)^n$$

$$a_N = a_0 + C^* \cdot \sum_{t=1}^{f} \left|R_t^2 - R_{t-1}^2\right|^n \cdot \sum_{B=1}^{N} \left(\frac{K_{ref,B}}{E}\right)^n$$

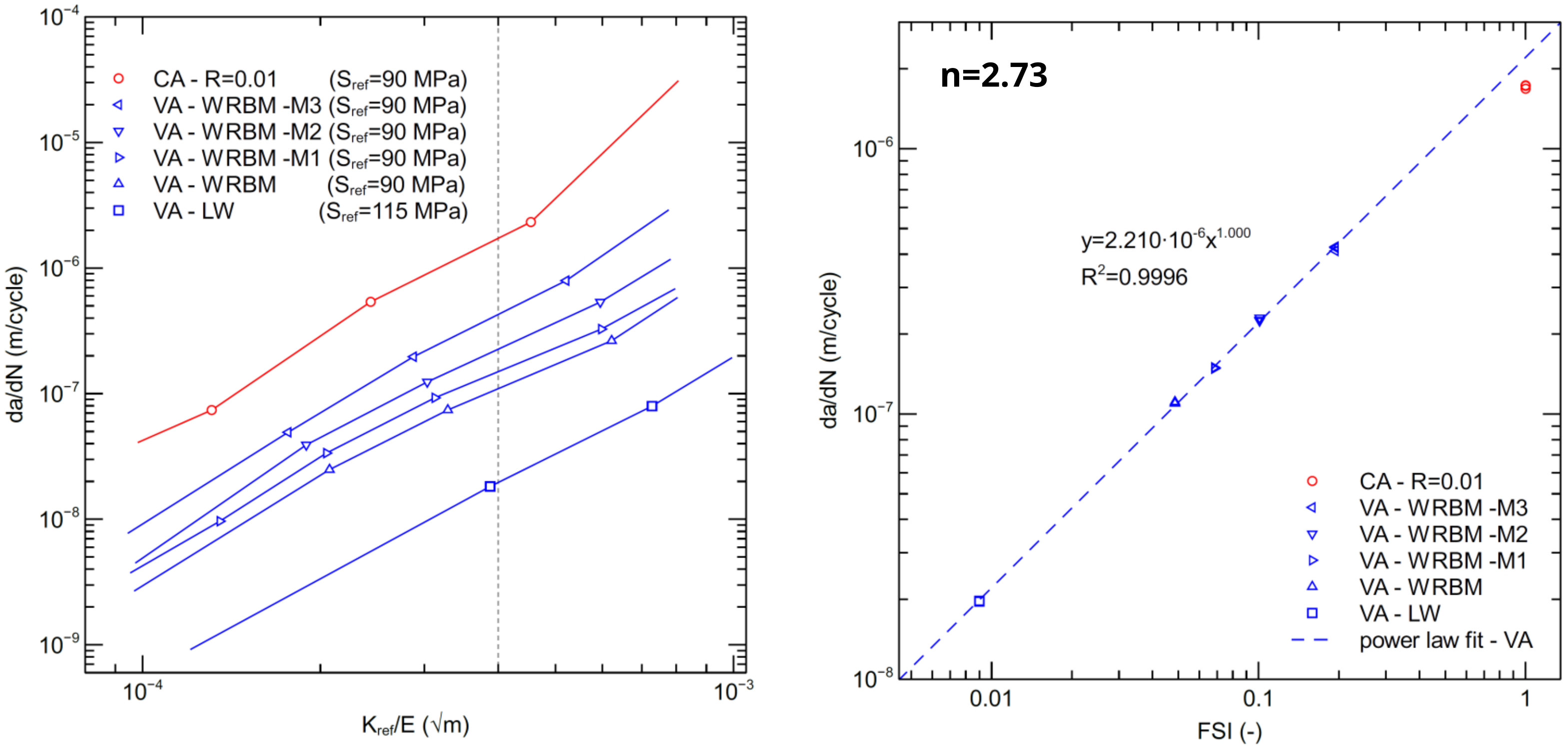
for limited crack growth in a spectrum block

$$FSI = \frac{1}{f}\sum_{t=1}^{f} \left|{R_t}^2 - {R_{t-1}}^2\right|^n$$

Fatigue Severity Index (FSI):
the average of the energy in the spectrum to the power *n*
(*FSI*=1 for CA FCGR with *R*=0)

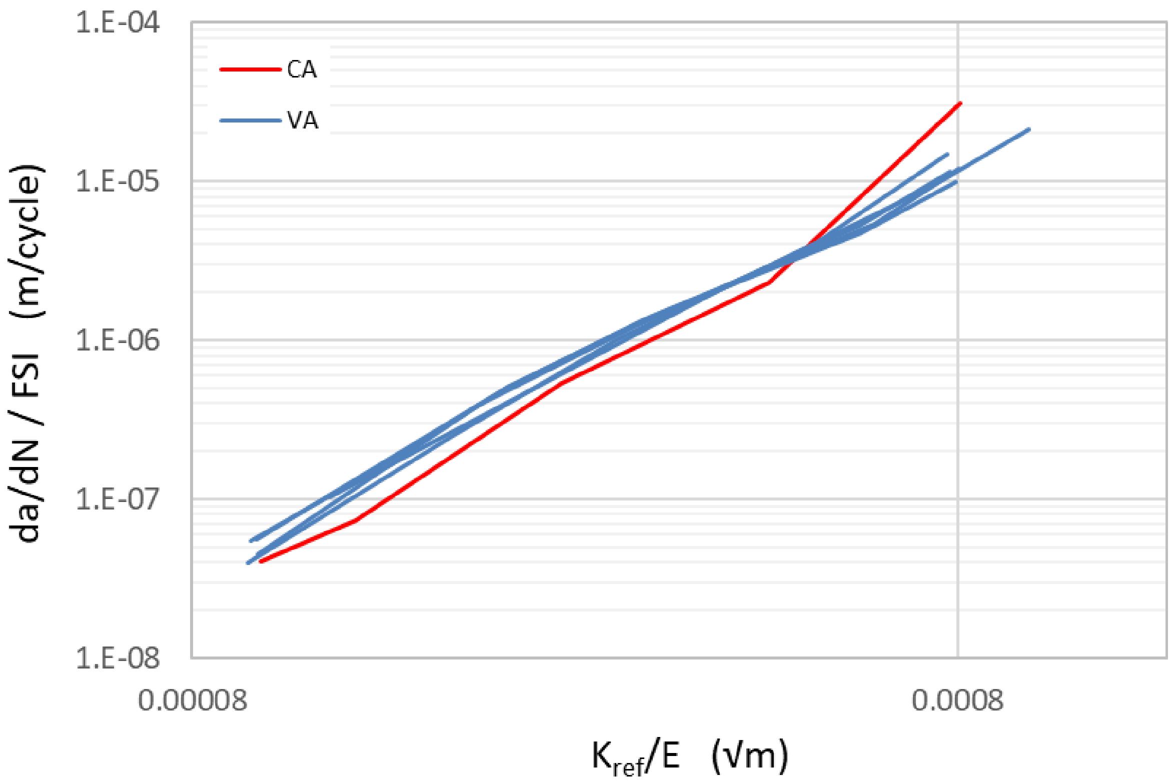
emiel.amsterdam@nlr.nl

Variable amplitude FCGR as a function of FSI

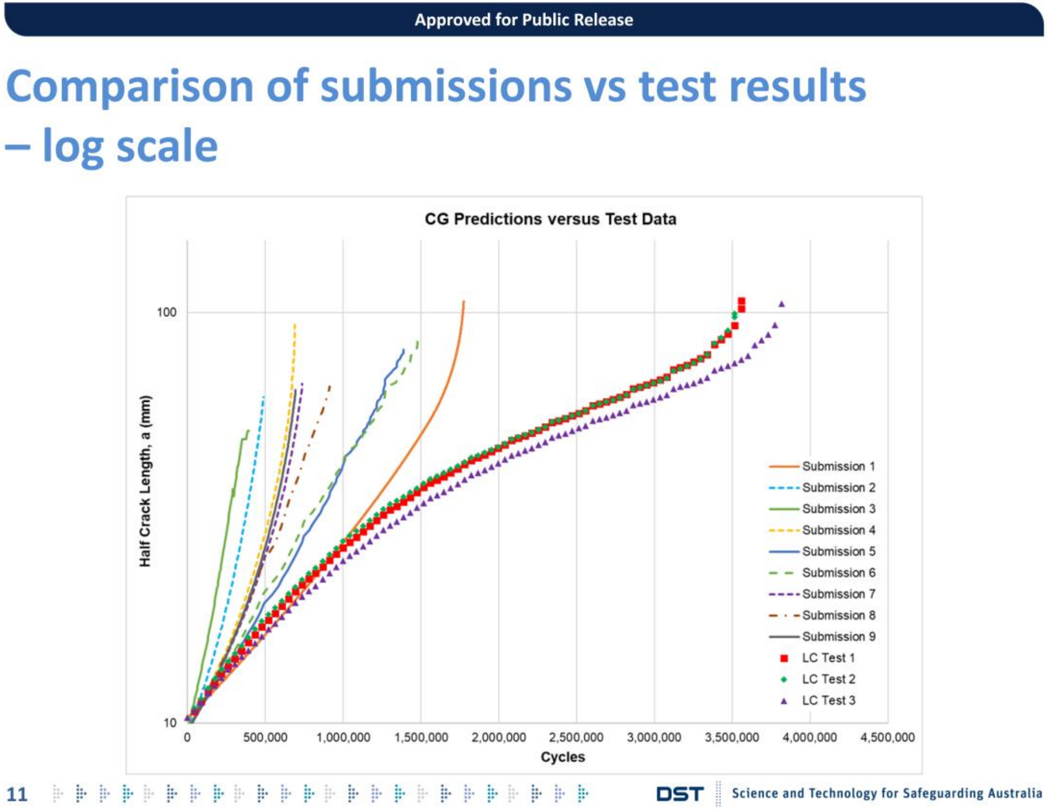


- WRBM spectrum with altered severities and lower wing skin spectrum
- 2.73 is the average slope of the constant amplitude FCGR curve

VA mastercurve



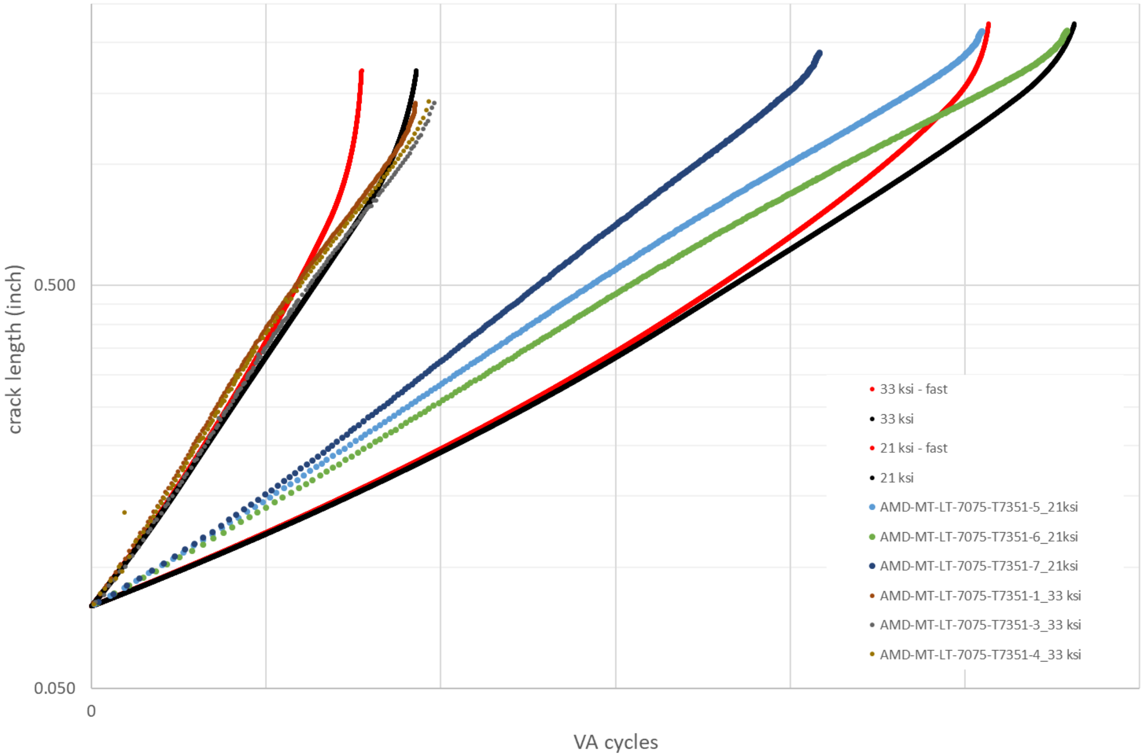
Blind prediction for ASSIST Challenge 2



http://www.meetingdata.utcdayton.com/agenda/asip/2020/proceedings/presentations/P20086.pdf

Blind prediction for USAF - A10 spectrum

Spectrum released and sent to NLR by Jacob Warner (USAF)



Conclusions

- We have proven our hypothesis that the parameter of similitude is proportional to the cyclic strain energy release rate (ΔG) and inversely proportional to the maximum stress intensity factor (K_{max}):

$$\frac{da}{dN} = C\left(\frac{\Delta G}{K_{max}}\right)^{n}$$

- The new parameter of similitude
  - Includes the original crack driving force
  - The effect of plasticity and variation of the plastic zone size with K_{max}
  - Gives a physical explanation for the inclusion of E
  - Gives a physical explanation for the stress ratio (mean stress) effect
  - Explains crack growth retardation under variable amplitude loading
    - No need for rainflow counting
    - Successfully applied for two blind predictions

E. Amsterdam, J.W.E. Wiegman, M. Nawijn, J.Th.M. De Hosson. *On the strain energy release rate and fatigue crack growth rate in metallic alloys*. Engineering Fracture Mechanics (2023) 109292.

Questions?

Available for free download till 26 June 2023:
https://authors.elsevier.com/a/1h2Wo38l3hjM7

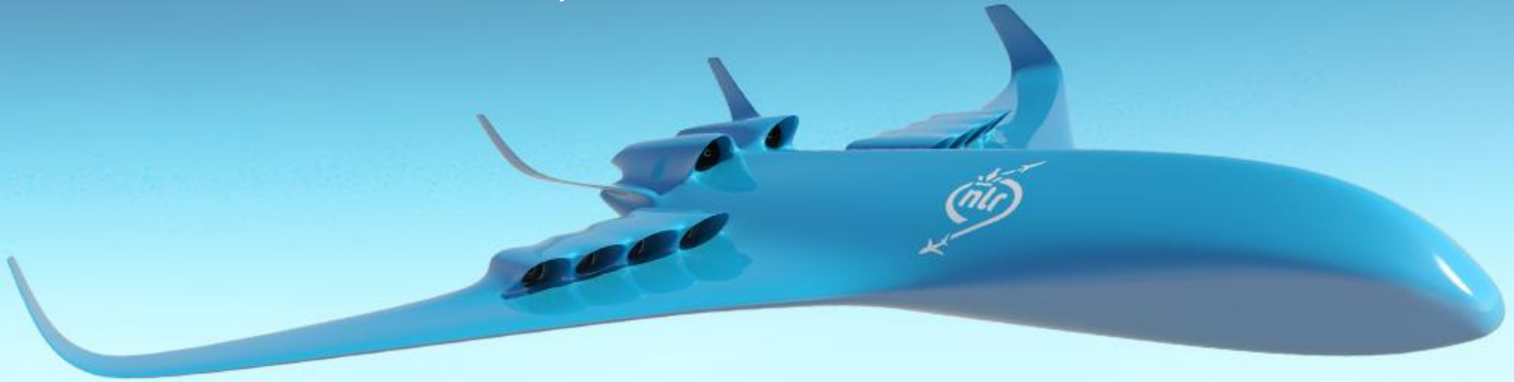
Contributions by all project partners are highly acknowledged

Royal Netherlands Air Force

Dedicated to innovation in aerospace

Fully engaged

NLR - Netherlands Aerospace Centre



Anthony Fokkerweg 2
1059 CM Amsterdam
The Netherlands

p) +31 88 511 31 13
e) info@nlr.nl i) www.nlr.org

Voorsterweg 31
8316 PR Marknesse
The Netherlands

p) +31 88 511 44 44
e) info@nlr.nl i) www.nlr.org